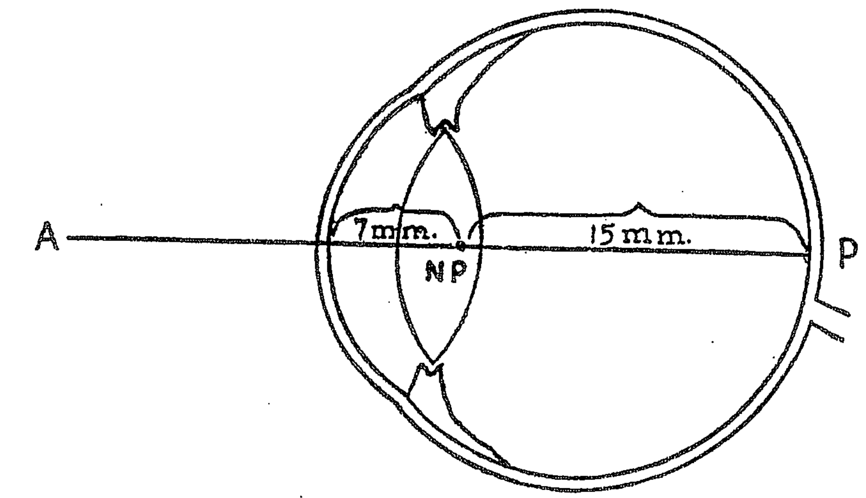
Optics of the Eye

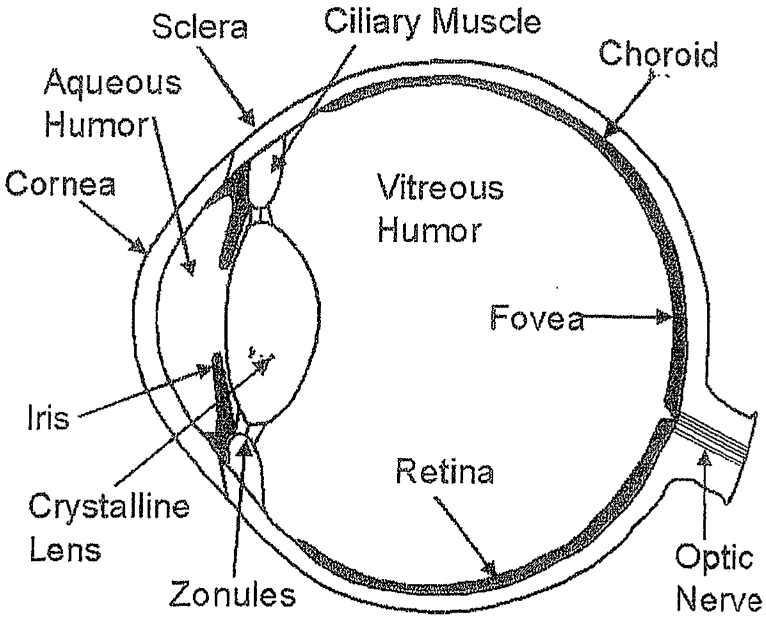
Ref. Meyer-Arendt, pp. 164-167

Hecht, pp. 201-210

J. Schwiegerling, Field Guide to Visual and Ophthalmic Optics (SPIE, 2004)



Eyeball



Top View of Right Eye

Cornea – Transparent membrane on the front of the eye. It contributes roughly two-thirds of the total power of the eye.
Aqueous Humor – Waterlike fluid in the anterior chamber between the cornea and the crystalline lens.
Iris – Pigmented diaphragm that is the eye's aperture stop.
Crystalline Lens – Gradient-refractive-index lens that changes shape to focus on near and distant objects. It contributes the remaining one-third power of the eye.
Vitreous Humor – Jellylike fluid in the posterior chamber between the crystalline lens and the retina.
Retina – Photosensitive surface of the interior of the eyeball that converts light to neural signals.
Fovea – The central, high-resolution portion of the retina.
Optic Disk – The "blind spot" where nerve fibers and blood vessels enter the eyeball.
Optic Nerve – The bundle of nerve fibers that carry the information from the retina to the brain.
Sclera – The "white" of the eye, which acts as a protective outer coating to the eyeball.
Choroid – An internal opaque membrane that absorbs stray light and provides structural support of the retina.

Optical Terms for Eye:

Far Point: farthest point where distinct vision is possible

Near Point: nearest point where distinct vision is possible

Emmetropic eye: normal eye where far point is infinity and near point is 25 cm

Ametropic eye: abnormal eye due to distortions in shape

Myopia: far point is nearer than infinity and near point is closer than 25 cm ("nearsightedness")

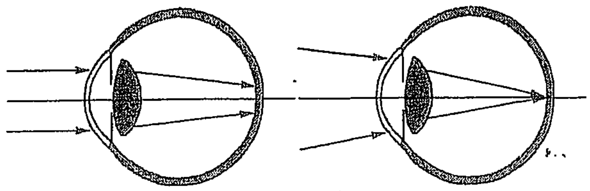
Hyperopia: near point is more distant than that of a normal eye ("farsightedness")

Presbyopia: recession of the near point with age as range of accommodation diminishes with age

Diopter: reciprocal of focal length of a lens. For example, 0.5 diopter lens has 2 meter focal length.

Angular magnification of lens = 25/f where f in cm

Accommodation: Fine focusing of the human eye performed by crystalline lens
(When the ciliary muscles are relaxed for the emmetropic eye, the light from an object at infinity will be focused on the retina)



Hyperopia

Presbyopia and hyperopia. The near point of either a presbyopic or a hyperopic eye is farther from the eye than normal. Then in order to see clearly an object at normal reading distance (this distance is usually assumed to be 25 cm or 10 inches), we must place in front of the eye a lens of such focal length that it forms an image of the object, at or beyond the near point. Thus the function of the lens is not to make the object appear larger, since the object and its image subtend equal angles at the lens, but in effect to move the object farther away from the eye to a point where a sharp retinal image can be formed.

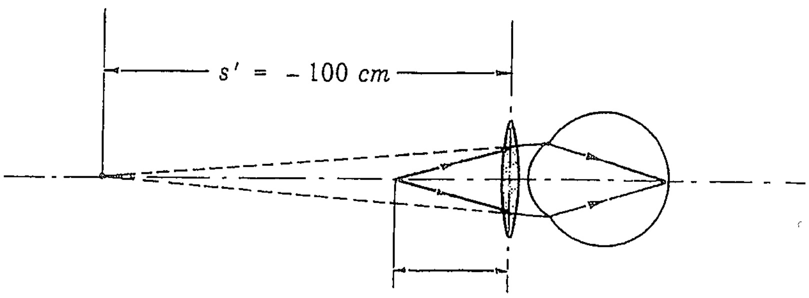


FIG. 6-6.

Example. The near point of a certain eye is 100 cm in front of the eye. What lens should be used to see clearly an object 25 cm in front of the eye? (See Fig. 6-6.)

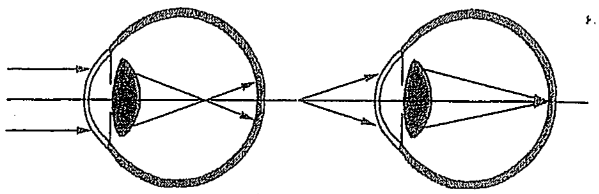
We have

$$s = +25 \text{ cm}, \quad s' = -100 \text{ cm}.$$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{+25} + \frac{1}{-100},$$

$$f = +33 \text{ cm}.$$

That is, a converging lens of focal length 33 cm is required.


Myopia

Myopia. The far point of a myopic eye is nearer than infinity. To see clearly objects beyond the far point, a lens must be used which will form an image of such objects, not farther from the eye than the far point.

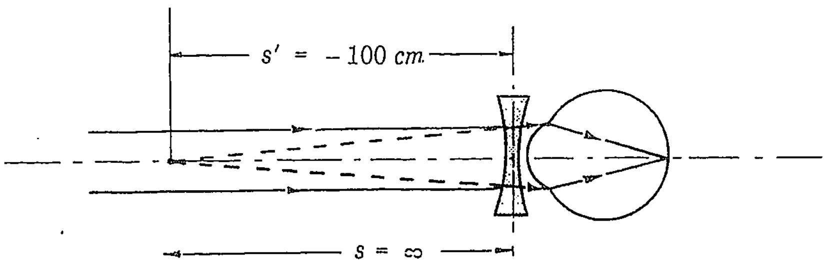


Fig. 6-7.

Example. The far point of a certain eye is 1 meter in front of the eye. What lens should be used to see clearly an object at infinity?
Assume the image to be formed at the far point. Then

$$s = \infty, \quad s' = -100 \text{ cm}.$$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-100},$$

$$\therefore f = -100 \text{ cm}.$$

A diverging lens of focal length 100 cm is required, as in Fig. 6-7.

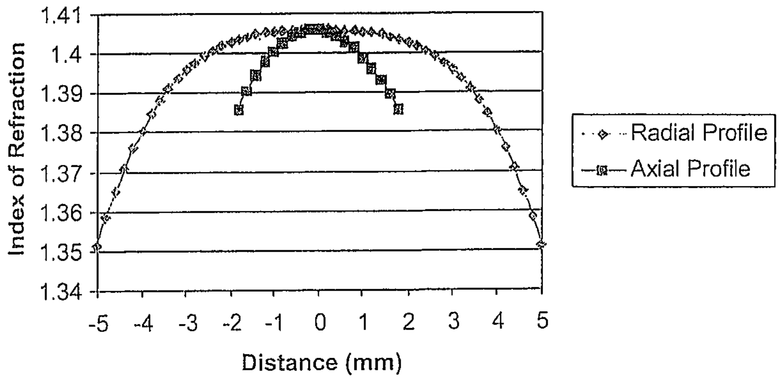
Properties of Ocular Components

Mean and Range

Anterior corneal radius: 7.80 mm (Range 7.00–8.65 mm)
Posterior corneal radius: 6.50 mm (Range 6.20–6.60 mm)
Anterior chamber depth: 3.68 mm (Range 2.80–4.60 mm)
Crystalline lens power: 20.35 D (Range 15.00–27.00 D)
Crystalline lens thickness: 4.00 mm
Anterior lens radius: 10.20 mm (Range 8.80–11.90 mm)
Posterior lens radius: 6.00 mm
Axial length: 24.00 mm (Range 20.00–29.50 mm)
Ocular power: 59.63 D (Range 54.00–65.00 D)

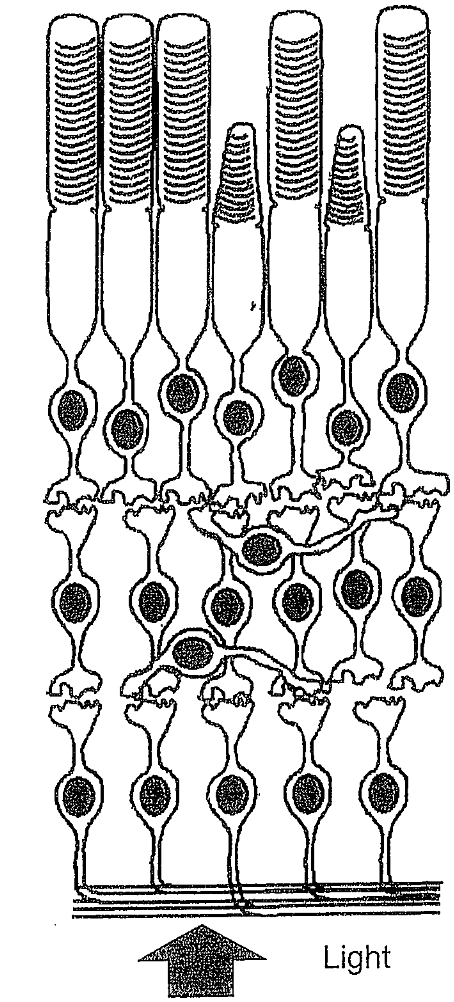
Material	Index	Abbe Number
Cornea	1.3771	57.1
Aqueous Humor	1.3374	61.3
Crystalline Lens	1.36 to 1.41	47.7
Vitreous Humor	1.336	61.1

The **crystalline lens** has a gradient index structure, such that its index of refraction varies both axially and radially. This distribution is not well documented *in vivo*, so good measures of the values across large populations do not currently exist. The **lens paradox** arises from a steepening of the lens surfaces with age, suggesting an increase in lens power, while the overall ocular power tends to reduce with age. The effective index of refraction of the crystalline lens must reduce with age, to account for the lens paradox.

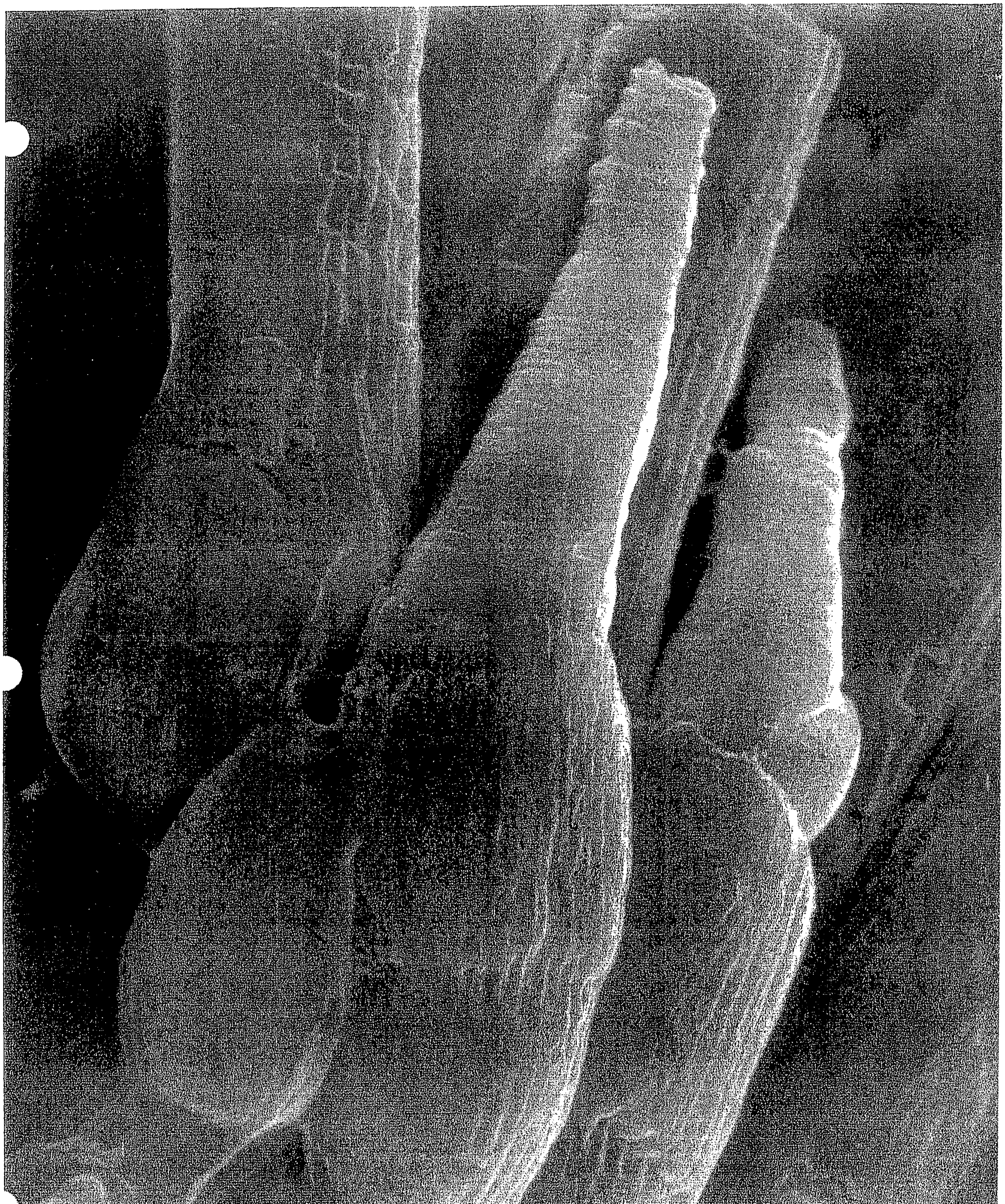


Name	Radius	Index	Thickness
	7.8 mm		
Cornea	6.5 mm	1.3771	0.55 mm
Aqueous	10.2 mm	1.3374	3.05 mm
Lens	-6.0 mm	1.4200	4.00 mm
Vitreous		1.3360	16.5966 mm

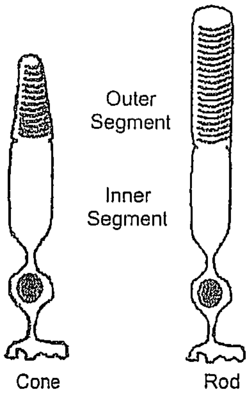
Retina



Light passes through multiple cell layers to reach the **photoreceptors**. Once absorbed, a signal is transmitted from the receptor through the **bipolar cells** to the **retinal ganglion cells**. From there, the signal propagates up into the brain for further processing. **Amacrine** and **horizontal cells** allow cells in a localized neighborhood to communicate with one another.



Photoreceptors



Two types of photoreceptors reside in the retina: **cones** and **rods**. The cones are responsible for daytime vision, while the rods respond under dark conditions. The cones come in three varieties: L, M, and S types (for long, middle, and short wavelength). Each cone type responds to a different portion of the visible spectrum, allowing for color vision. Rods have a spectral sensitivity that differs from the cones. Photoreceptors are specialized cells for detecting light. They are composed of the **outer nuclear** layer that contains the cell nuclei, the **inner segment** that houses the cell machinery, and the **outer segment** that contains photosensitive pigment. The outer segment of a rod has discrete disks saturated with rhodopsin molecules, while the outer segment of a cone contains similar photosensitive molecules in a series of folds. The outer segment absorbs photons, which initiates an electrochemical transmission through the cells and retinal nerve fibers, up into the brain.

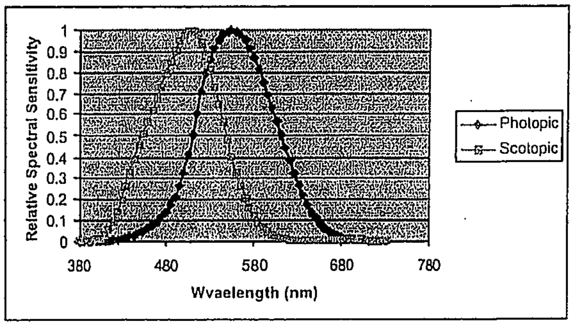
Cones	Rods
Color Vision	Monochromatic
No sensitivity in the dark	High sensitivity in the dark
Respond in bright light	Bleached in bright light
Slow temporal response	Fast temporal response
Mostly in fovea	Mostly in periphery
Some in peripheral retina	None in fovea
High visual acuity	Low visual acuity
In fovea, one neuron per cone	Many rods per single neuron

Cone diameter is roughly 2.5 μm in the fovea and rapidly increases outside fovea to 10 μm in periphery. Rod diameter is roughly 3 μm at a field angle of 18° and increases in size to 5.5 μm in periphery. The central 200 μm of the retina is free of rods. The total number of cones in the retina is 6.4 million. There are roughly 125 million rods in the retina.

Color blindness is abnormal color vision associated with one or more cone types. Protanopia and deutranopia are sometimes called red/green color blindness since these colors are difficult to distinguish in these individuals. Tritanopia is sometimes called blue/yellow color blindness, since individuals with this deficit confuse these colors.

Type	Missing	Prevalence (Male/Female)
Protanopia	L Cones	1% M / Rare F
Deutranopia	M Cones	1% M / 0.01% F
Tritanopia	S Cones	Rare MF

Schematic Eyes - Spectral Sensitivity



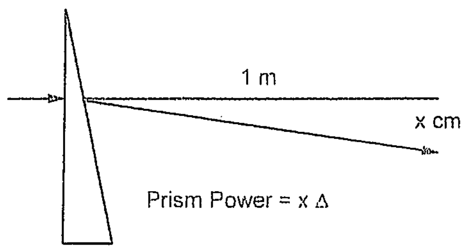
Scotopic - Low light level
Peak around 505 nm

Photopic - High light level
Peak around 555 nm

Mesopic - In between

To include in raytracing code, weight wavelengths by appropriate curve.

Prismatic Error

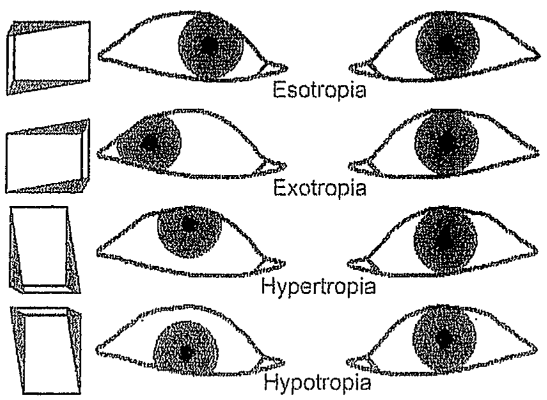


Prisms and prismatic error in ophthalmic optics are specified in terms of prism diopters (Δ). The definition of a **prism diopter** is the prismatic deviation of a beam of light 1 cm at a distance of 1 m. The orientation of a prism is defined by its base (the wide end of the prism). Base Up (BU) and Base Down (BD) cause vertical deviations, while Base Out (BO) and Base In (BI) cause horizontal deviations. Base Out orients the prism towards the temple, while Base In orients the prism nasally.

Decentering a spectacle lens introduces prismatic effects. **Prentice's Rule** describes the amount of prism P introduced in this situation. For a lens of power ϕ diopters and a decentration d in cm, the prismatic effect is given by

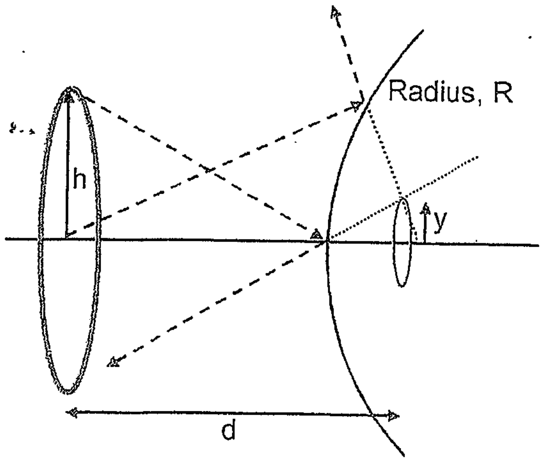
$$P\,(\Delta) = d\,(\text{cm}) \times \phi\,(\text{D})$$

Prism power is used to alleviate binocular alignment errors. **Strabismus** is a condition where the line of sight of the two eyes does not meet at the fixation point. Different types of deviations occur in strabismus. Prisms with the appropriate orientation are used to correct the misalignment. In cases of large misalignment, the prism may be split between the two eyes. In this case, equal-power prisms are placed over each eye with their bases in opposite directions. If refractive error and strabismus are present, then spectacle lens decentration introduces a prismatic correction. Strabismus is a serious condition if left untreated in young children. **Amblyopia**, or permanent loss of acuity can result from the misalignment, unless corrected in the first few years of life.



Keratometry

Keratometry is the measurement of the corneal radius of curvature. The anterior corneal surface is treated as a *specular* reflector. A ring of known size is placed in front of the eye. The cornea (in reflection) forms a virtual image of the ring below its surface. The virtual image is the first Purkinje image of the ring. The size of this image is related to the radius of curvature, R, of the cornea by $R = 2dy / h$, where h is the radius of the ring object, y is the radius of the ring image, and d is the distance between the object and image. In converting the corneal radius to corneal power it is customary to use the **keratometric index of refraction** n_k = 1.3375, instead of the actual index of refraction of the cornea. The keratometric index is an effective index that accounts for the negative power introduced by the posterior corneal surface. Consequently, keratometry attempts to predict the total corneal power based only on a measurement of the anterior corneal surface. The corneal power in diopters is given by $\Phi = 337.5 / R$, for R in mm.



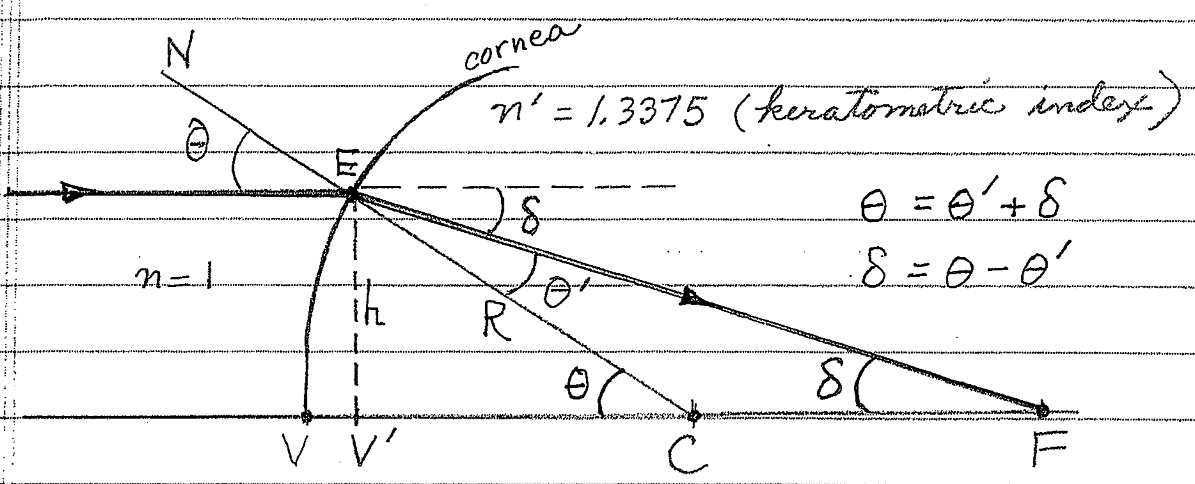
When corneal astigmatism is present, the ring image becomes elliptical. The major and minor axes of this ellipse define the orientati of the corneal astigmatism. In this case, keratometry is normally measured along these two orthogonal meridians, giving the maximum and minimum corneal power. These extrema are known as **K-values** or **corneal Ks**. The difference between the K-values is a measure of corneal astigmatism. **With-the-rule astigmatism** is when the vertical meridian is steepest (smallest radius). **Against-the-rule astigmatism** is the converse case. Irregularities in corneal shape cause further distortion of the ring image, providing a subjectively assessment of corneal shape.

Reduced Eye Model

Only considers front of cornea

(Not paraxial in general) ..

$R = 8$ mm



$\sin\theta = n' \sin\theta'$

$\sin\theta = h/R$

$\sin\theta' = \dfrac{h}{n' R}$

Call $R = 8$ mm

$\overline{V'F} = a$

$\tan\delta = h/a$

$\overline{VV'} = d$, $\overline{V'C} = b$

$a = h/\tan\delta$

$\overline{VF} = f$

$b = R\cos\theta$

$d = R - b = R(1-\cos\theta)$

h	θ	θ'	δ	a	d	f
1 mm	7.18°	5.36°	1.817°	31.523 mm	0.063 mm	31.586 mm
3 mm	22.024°	16.2825°	5.74°	29.845	0.584	30.429

Spectacle and Contact Lens Materials

Material or Manufacturer	Refractive Index (d)	Abbe Number	Specific Gravity
Ophthalmic Crown	1.523*	58.9	2.54
CR-39	1.498	58.0	1.32
Polycarbonate	1.586	30.0	1.20
Seiko	1.740	33.0	1.47
Seiko Super 1.6	1.600	42.0	1.22
Seiko	1.670	32.0	1.36
Seiko	1.560	40.0	1.17
Zeiss Claret	1.600	36.0	1.34
Sola Spectralite	1.537	47.0	1.21
Essilor 17	1.701*	42.0	3.21
Essilor 16	1.600*	42.0	2.63
Essilor 18	1.802*	35.0	3.65
Essilor Ormex	1.557	37.0	1.23
Essilor Orma	1.500	59.0	1.32
Sola Finalite	1.600	42.0	1.23
Sola 1.66	1.660	32.0	1.35
Sola Glass White	1.523*	59.0	2.62

*Glass

Spectacle lenses are made from glass and plastic materials. A table of some common materials is shown above. High index material is desirable to keep the lenses thinner (lighter) and high Abbe numbers are desirable to minimize chromatic aberration. Glass materials tend to be much heavier than plastic. ANSI standards require that lenses withstand the impact of a 1-inch steel ball dropped from a height of 50 inches without fracturing. Safety glasses must withstand the impact of a 0.25-inch steel ball traveling at 150 ft/s.

Blind Spot

At the point where the optic nerve enters the eye, the retina is insensitive to light and is called the blind spot. In the eyeball, the blind spot is a short distance from the fovea (where vision is the most distinct) toward the nasal side. Then it is possible that with either eye an object to one side of that on which attention is fixed may be unseen (disappear), provided the observer is the proper distance away.

If one closes the right eye while the figure is held an appropriate distance away (about 6 inches), spot A will disappear when attention is fixed on B. With the left eye closed, spot B will disappear when attention is fixed on A.

This can be done with many two point objects provided their angular separation is about 22° to 25°. This can be demonstrated nicely with two point lights when the observer is at the appropriate distance to achieve the proper angular separation.

Edmé Mariotte's discovery of the blind spot (1668)

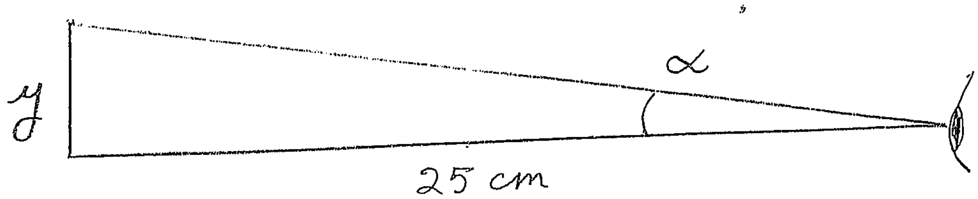
In 1668, a small book of 27 pages was published in Paris, bearing the title: "Nouvelle Découvverte touchant la veüe". This was the first public description ever of the blind spot in the inner eye. The book contains a letter by Edmé Mariotte to Jean Pecquet, followed by a reply from the latter.

Mariotte described a well-prepared experiment he had performed. His plan had been to find out, whether perception would be strongest at the spot where the optical nerve enters the eye. Leonardo da Vinci had said so, and according to Cartesian physiology perception was nothing but transport of movement by the nerves. So vision should be strongest where the nerve was directly hit by moving particles carrying the optical information. In order to project an image exactly on to that spot, Mariotte fixed two white circles on a dark background, separated by a distance of two feet. Closing his left eye, he watched the left circle with his right eye from a short distance. Then he moved backwards, and at a certain point the second paper to the right disappeared completely ("me disparu entièrement"). This loss of vision ("default de vision") with one eye opened wide was a shocking experience to him. At first he suspected that someone might have taken the second circle away. But it was untouched and returned into view as soon as he moved forward or backward. So this loss of vision was depended on the position of the eye. Mariotte was well aware that he had found what he was looking for: the entrance of the optical nerve. The amazing fact was that perception here wasn't strongest, but completely missing. The very center of the organ of sight is blind - which leads to the following remark: "I see I'm blind."

The experiment became popular in the Paris scientific community, especially in the Academie des Sciences, founded in 1666 (Mariotte and Pecquet were founding members). Mariotte demonstrated the blind-spot in the *King's Library*, the later Bibliothèque Nationale. According to Hermann von Helmholtz, he was even invited by the King of England to demonstrate his discovery. Astronomers were interested as well (as Pecquet mentions in his letter), for they often lost sight of stars while watching others and Mariotte seemed to offer a rational explanation for that by testing the physiology of the eye.

How can we be unaware of such a large defect in the visual field (typically about 5°–8°)? The optic disk is located in the nasal retina of each eye. With both eyes open, information about the corresponding region of visual space is, of course, available from the temporal retina of the other eye. But this fact does not explain why the blind spot remains undetected with one eye closed. When the world is viewed monocularly, the visual system appears to "fill-in" the missing part of the scene based on the information supplied by the regions surrounding the optic disk. To observe this phenomenon, notice what happens when a pencil or some other object lies *across* the optic disk representation. Remarkably, the pencil looks complete! Although electrophysiological recordings have shown that neurons in the visual cortex whose receptive fields lie in the optic disk representation can be activated by stimulating the regions that surround the optic disk of the contralateral eye, suggesting that "filling-in" the blind spot is based on cortical mechanisms that integrate information from different points in the visual field, the mechanism of this striking phenomenon is not clear.

Angular Magnification



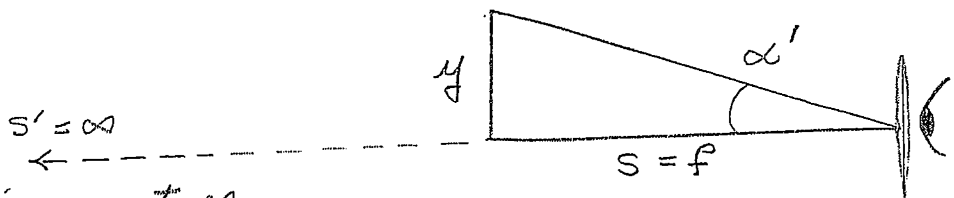


image at ∞

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, \quad \frac{1}{s} + \frac{1}{\infty} = \frac{1}{f}$$

$$\gamma = \frac{\tan \alpha'}{\tan \alpha} = \frac{y/f}{y/25\,\text{cm}} = \frac{25\,\text{cm}}{f}$$

Aberrations limit γ to about 3 ×

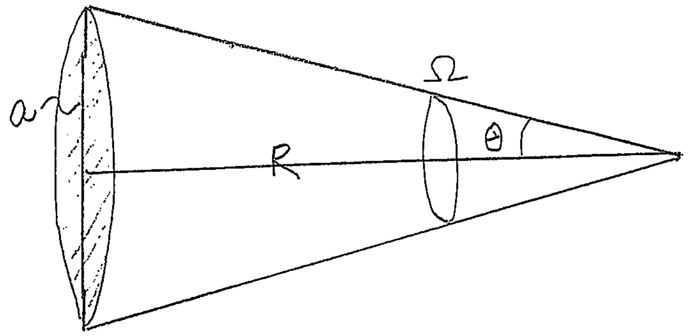
SPEED OF LENSES

Light gathering power of lens

focal ratio (f-number) $\equiv \frac{f}{a}$

where $a \equiv$ diameter of stop

speed $\sim$ $(f/\text{number})^{-2}$



solid angle $\Omega \equiv \frac{A}{R^2}$ where A = area subtended

from geometry, $A = 2\pi R a$

$a = R(1-\cos\theta)$

$A = 2\pi R^2(1-\cos\theta)$

So $\Omega = 2\pi(1-\cos\theta)$

If θ small, $\cos\theta \sim 1 - \frac{\theta^2}{2}$

Then $\Omega \cong \pi\theta^2$

From trig, $\tan\theta \simeq \frac{a/2}{f}$ $(R \equiv f)$

Illumination $\sim \Omega \simeq \frac{\pi a^2}{4f^2}$

Thus $\boxed{\text{speed} \sim \frac{1}{\left(\frac{f}{a}\right)^2}}$

Exposure time increases with square of f/number

Relative Aperture and f-Number

Suppose we collect the light from an extended source and form an image of it using a lens (or mirror). The amount of energy gathered by the lens (or mirror) from some small region of a distant source will be directly proportional to the area of the lens or, more generally, to the area of the entrance pupil. A large *clear aperture* will intersect a large cone of rays. Obviously, if the source were a laser with a very narrow beam, this would not necessarily be true. If we neglect losses due to reflection, absorption, and so forth, the incoming energy will be spread across a corresponding region of the image (Fig. 5.38). The energy per unit area per unit time (i.e., the flux density or irradiance) will be inversely proportional to the image area.

The entrance pupil area, if circular, varies as the square of its radius and is therefore proportional to the square of its diameter D. Furthermore, the image area will vary as the square of its lateral dimension, which in turn [Eqs. (5.24) and (5.26)] is proportional to f^2. (Keep in mind that we are talking about an extended object rather than a point source. In the latter case, the image would be confined to a very small area independent of f.) Thus the flux density at the image plane varies as $(D/f)^2$. The ratio D/f is known as the *relative aperture*, and its inverse is the **focal ratio** or ***f*-number**, often written $f/\#$, that is,

$$f/\# \equiv \frac{f}{D} \tag{5.40}$$

where $f/\#$ should be understood as a single symbol. For example, a lens with a 25-mm aperture and a 50-mm focal length has an *f-number* of 2, which is usually designated $f/2$. Figure 5.39 illustrates the point by showing a thin lens behind a variable iris diaphragm operating at either $f/2$ or $f/4$. A smaller *f-number* clearly permits more light to reach the image plane.

Camera lenses are usually specified by their focal lengths and largest possible apertures; for example, you might see "50 mm, $f/1.4$" on the barrel of a lens. Since the photographic exposure time is proportional to the square of the *f-number*, the latter is sometimes spoken of as the **speed** of the lens. An $f/1.4$ lens is said to be twice as fast as an $f/2$ lens. Usually, lens diaphragms have *f-number* markings of 1, 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22, and so on. The largest relative aperture in this case corresponds to $f/1$, and that's a fast lens—$f/2$ is more typical. Each consecutive diaphragm setting increases the *f-number* by a multiplicative factor of $\sqrt{2}$ (numerically rounded off). This corresponds to a decrease in relative aperture by a multiplicative factor of $1/\sqrt{2}$ and therefore a decrease in flux density by one half. Thus, the same amount of light will reach the film whether the camera is set for $f/1.4$ at 1/500th of a second, $f/2$ at 1/250th of a second, or $f/2.8$ at 1/125th of a second.

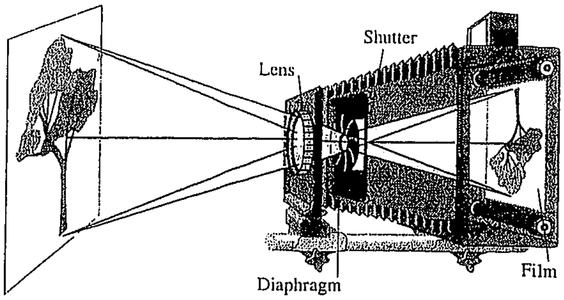


Figure 5.38 A large-format camera usually consists of a lens, followed by an adjustable diaphragm, a shutter that can rapidly open and close, and a sheet of film on which the image is formed.

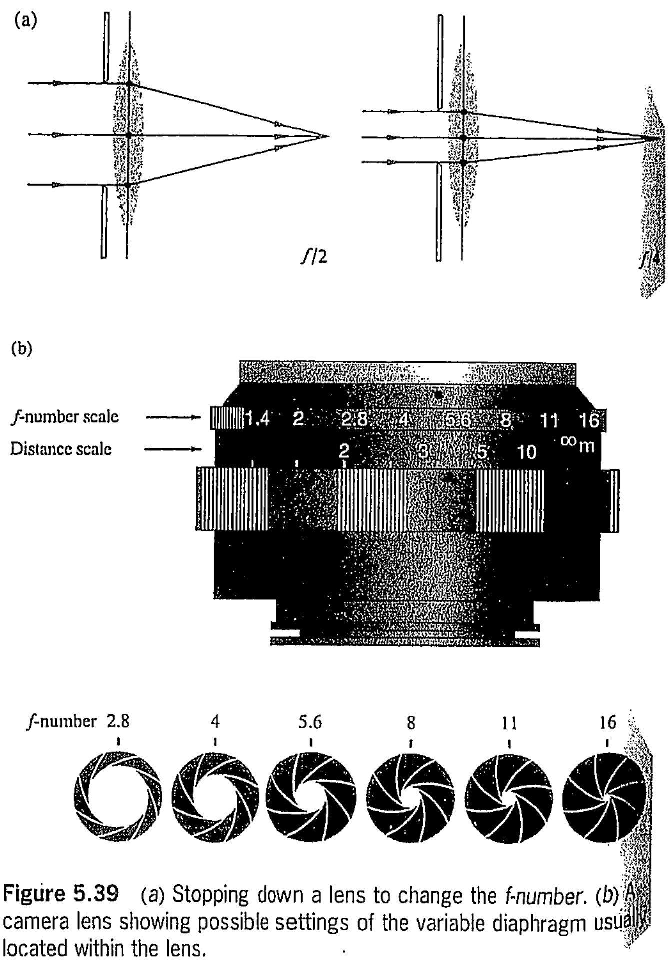


Figure 5.39 (*a*) Stopping down a lens to change the *f-number*. (*b*) A camera lens showing possible settings of the variable diaphragm usually located within the lens.

$f/\#$	1.4	2	2.8	4	5.6	8	11	16
$(f/\#)^2$	1.96	4	7.84	16	31.36	64	121	256