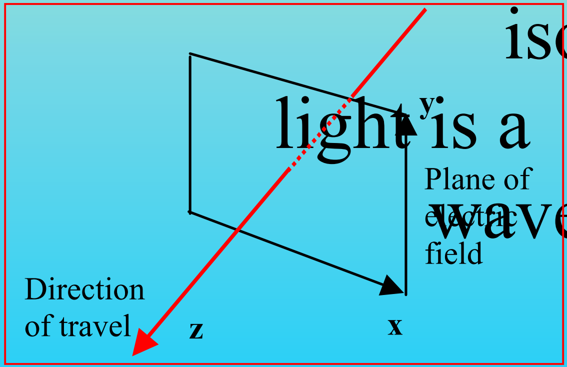
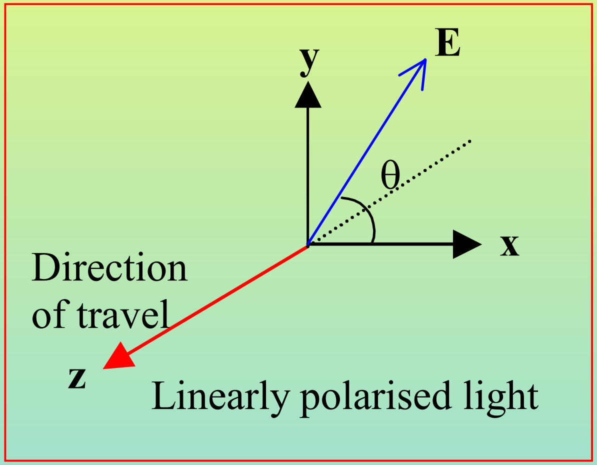
Polarisation of light

- The polarisation of light is scarcely discernable with our eyes
- Polarisation describes the behaviour of the electric field associated with light
  - types of polarisation are **linear**, **elliptical**, **circular**, **unpolarised**
- Remember that in isotropic materials, light is a transverse wave



Linear polarisation

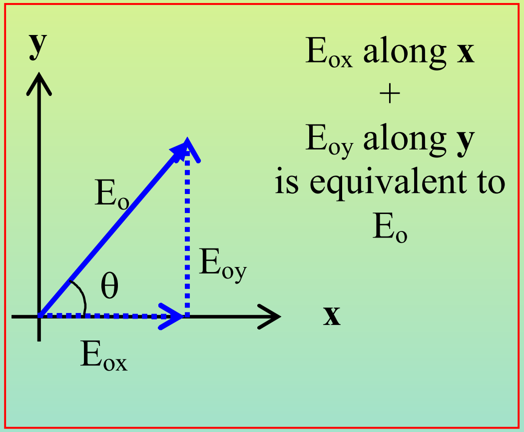
- The direction of the electric field at a point stays constant in time
  - its direction is the **direction of linear polarisation**
  - its components along the x and y axes must always stay in step
  - mathematically, the 2 components of **E** at point z along the wave can be written



$$\boxed{\begin{aligned} E_x(z,t) &= E_{ox}\cos(kz-\omega t) \\ E_y(z,t) &= E_{oy}\cos(kz-\omega t) \end{aligned}}$$

A note on components of **E**

- **E**, the electric field, has a direction and a size
  - it is a **vector**, like a displacement



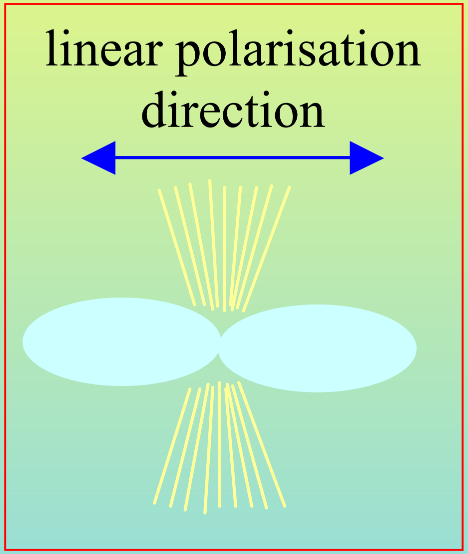
E_{ox} along **x**
+
E_{oy} along **y**
is equivalent to
E_o

- Every electric field of magnitude E_o has **components,** E_{ox} and E_{oy}
  - the sizes of the components depend on the angle θ between E_o and the **x** axis

$$\boxed{\begin{aligned} E_{ox} &= E_o \cos(\theta) \\ E_{oy} &= E_o \sin(\theta) \end{aligned}}$$

- Polaroid transmits the component of **E** along its axis (see later)

Haidinger's brush



- Some people can detect the direction of linear polarisation of light
- A very faint figure is visible in linearly polarised light a few degrees across in the centre of your field of view
  - if you rotate a piece of polaroid in front of your eye, this figure rotates with the polaroid
- The figure is called **Haidinger's brush**

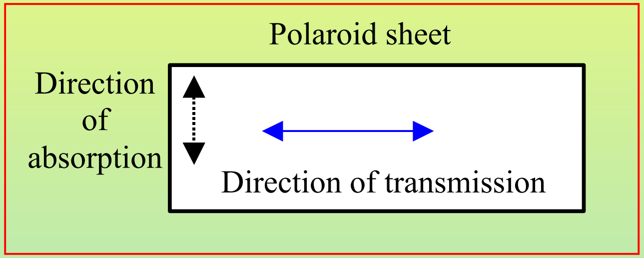
Relationship between irradiance of light and electric field **E**

- Light meters measure irradiance, cameras and our eyes respond to irradiance
- The irradiance, I, is proportional the average square of the electric field:

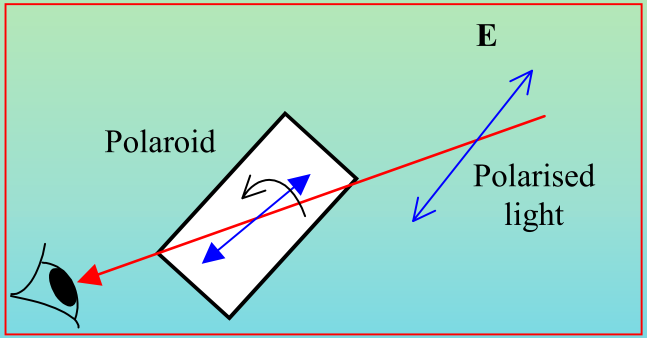
$$\boxed{I \propto \langle E^2 \rangle}$$

- Polarisation phenomena are about the direction and amplitude of the electric field wave, **E**

Polaroid sheet

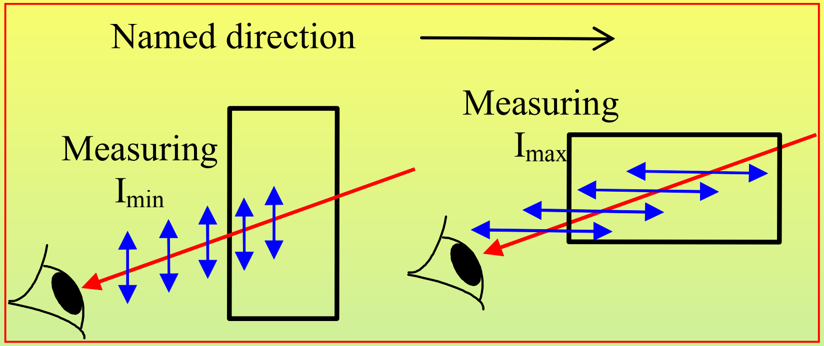


- Polaroid produces linear polarisation of light by transmitting the electric vector along the axis of the polaroid and absorbing the perpendicular electric vector



- Polaroid placed in front of polarised light transmits the most when its axis is rotated || to the direction of polarisation and least when ⊥

% of polarisation



- Light can be partially polarised
- Measure the maximum intensity I_{max} and the minimum intensity I_{min}
- Calculate the % polarisation in the direction of maximum intensity

$$\boxed{\%\ polarisation = \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100}$$

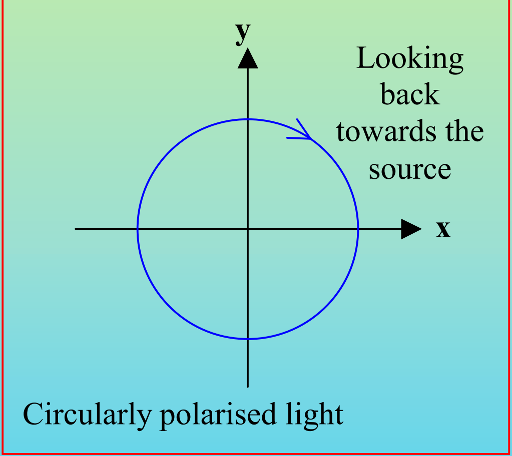
- Example:
  - if $I_{max} = 2I_{min}$, then % polarisation = 100/3 = 33%

Circular polarisation

- With circular polarisation, the **x** and **y** amplitudes are both equal (call them E_o) but there is a phase difference of π/2 between them
- Circular polarisation comes in two flavours
  - **right circular** polarisation, in which **E** rotates clockwise looking back down along the direction of propagation

$$\boxed{\begin{aligned} E_x &= E_o \cos(kz - \omega t) \\ E_y &= E_o \sin(kz - \omega t) \end{aligned}}$$

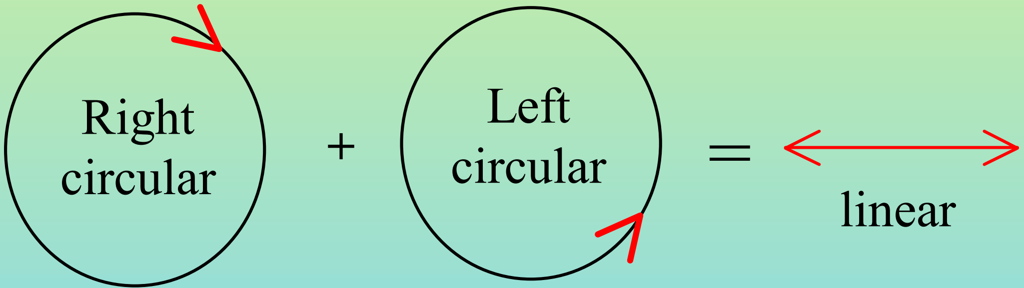
  - left-hand circular polarisation
    - circular polarisation can't be distinguished through a sheet of polaroid



Circularly polarised light

Combination of opposite circular polarisations

- If you combine right-handed and left-handed circular polarisation in equal amounts, you get linear polarisation



$$E_x = E_o \cos(kz - \omega t) \qquad E_y = E_o \sin(kz - \omega t)$$

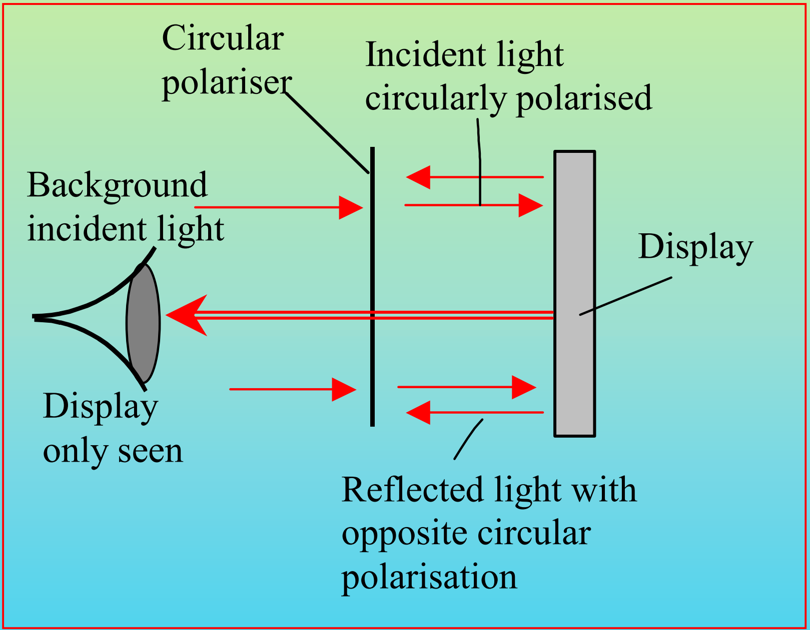
$$E_x = E_o \cos(kz - \omega t) \qquad E_y = -E_o \sin(kz - \omega t)$$

$$E_x = 2E_o \cos(kz - \omega t)$$

- The polarisation angle (i.e. the direction of the linear polarisation) depends on the phase difference between one component (e.g. *x* component) of the two hands
    - relevant to interpreting other polarisation phenomena

Application of circular polarisation

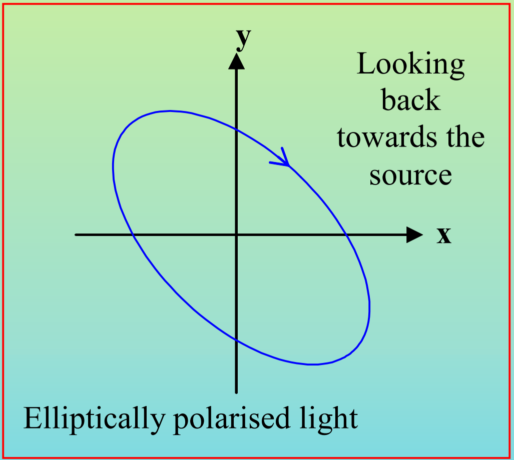
- Circular polarisers are used to enhance the contrast of LED displays
- Background light is circularly polarised before it reaches the reflecting front of the display
- The handedness of the polarisation is changed by the reflection and it fails to get back through the polariser
- The direct light from the display does pass through the polariser



Elliptically polarised light

- With elliptical polarisation, the amplitudes of **x** and **y** components are generally not equal and neither are phases between the components anything special

$$\boxed{\begin{aligned} E_x &= E_{ox}\cos(kz-\omega t) \\ E_y &= E_{oy}\cos(kz-\omega t+\varepsilon) \end{aligned}}$$

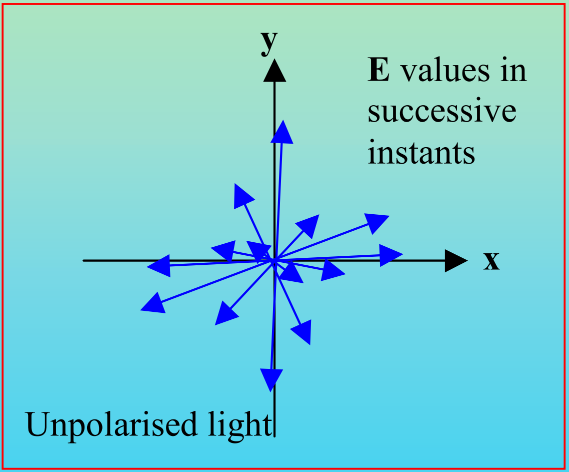


Elliptically polarised light

- Elliptical polarisation is the most general case
  - ε = 0 is the special case of linearly polarised light
  - ε = ±π/2 **and** $E_{oy} = E_{ox}$ gives circularly polarised light

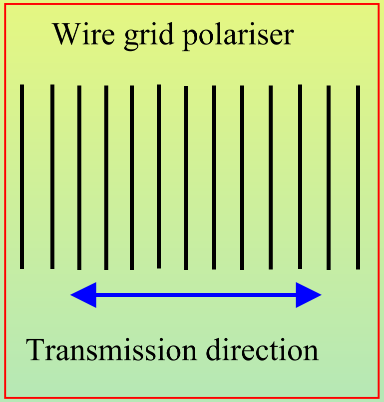
Unpolarised light

- Unpolarised light consists of light where the direction of **E** varies at random between successive measurements at one point
  - any direction is equally likely
- Unpolarised light can be considered as a combination of equal amounts of linear polarisation in two directions at right angles, where the **two components are incoherent**



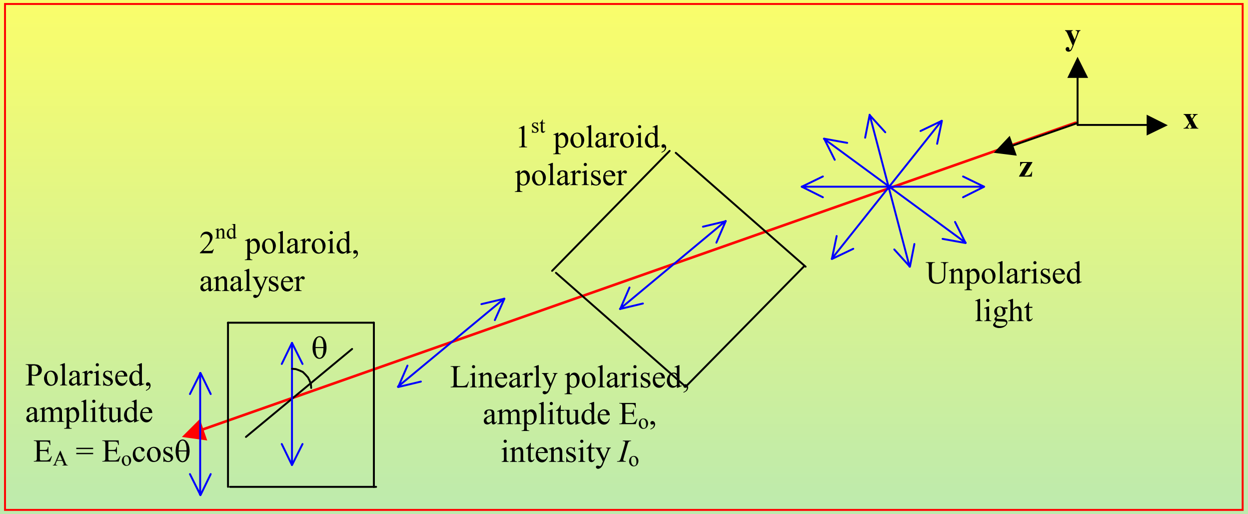
Producing linear polarisation

- Polaroid sheet
- Transmission through a wire grid
  - the distance between wires < λ/4
    - modern polaroid sheet works in a similar way



- Scattering of sunlight by the atmosphere
  - bees and other insects use polarised light to navigate
- Reflecting light
  - reflections can be reduced by looking though polaroid sunglasses oriented to cut out the strongest polarisation
- Transmission through birefringent materials
  - used in the petrological microscope
  - analysis of strain in transparent materials

Malus' law



- Malus' law gives the irradiance transmitted by an analysing polariser, I_A, set at angle θ to the direction of polarised light of irradiance I_o
- The irradiance of the light transmitted varies as $\cos^2\theta$
  - this is just what you'd expect from our earlier section on the relationship between irradiance and amplitude

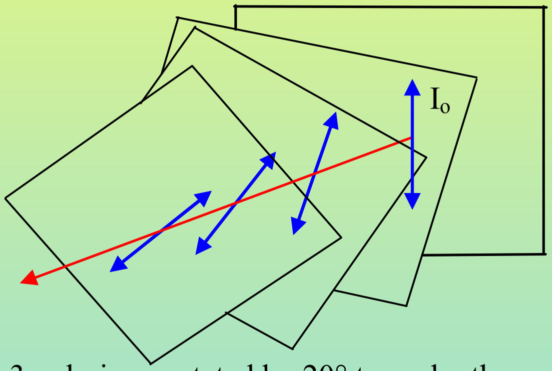
$$\boxed{I_A = I_0 \cos^2\theta}$$

  - e.g. a polariser is set at 30° to the direction of polarised light, *how much is transmitted by the polariser?*
    - fraction transmitted = 0.75

$$\boxed{I_A = I_o \cos^2(30°) = 0.75 I_o}$$

Rotating the direction of polarisation

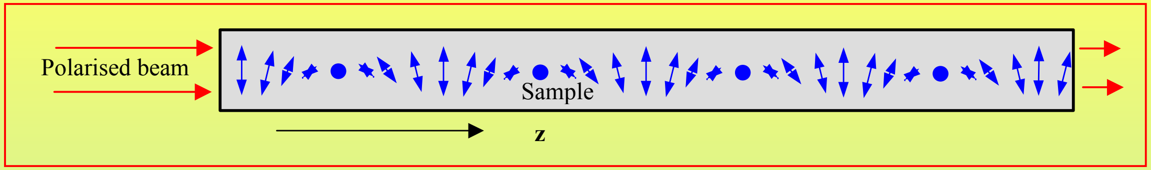
- Several sheets of polaroid in succession will rotate the direction of polarisation of light



3 polarisers rotated by 20° to each other

- Some molecules, such as sugar solutions and quartz, can do the same only more efficiently. This ability is called **optical activity**, or sometimes rotary polarisation

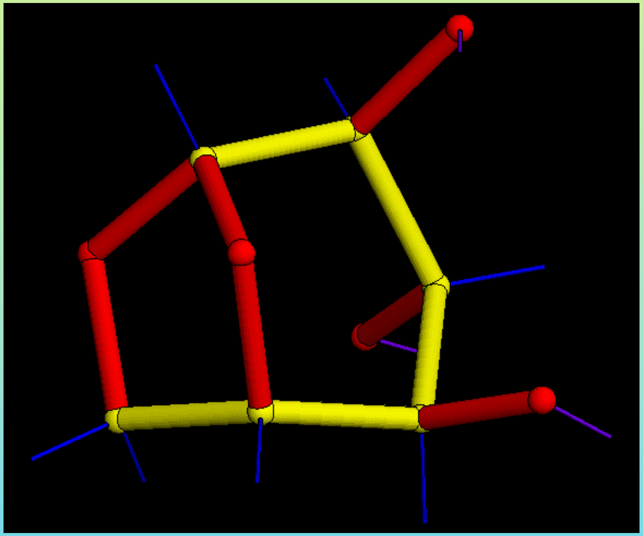
Optical activity



- Optically active materials rotate the direction of polarisation as the light propagates through
  - **dextro-rotatory**; **levo-rotatory**
  - measured by **specific rotation**, in ° mm^{-1} for solids
- Cause is that left and right circularly polarised light have different refractive indices n_R and n_L.
  - linearly polarised light travels through as two circularly polarised rays, at slightly different speeds
    - as their phase difference varies, so the direction of linear polarisation alters

Chiral molecules

- Optical activity is caused by molecules that have a helical twist, called **chiral** molecules
- All chiral amino acids are l-rotatory – why?
- Natural sugars like dextrose are d-rotatory
- (Some optical activity can be caused by twisted molecular arrangements)



Dextrose

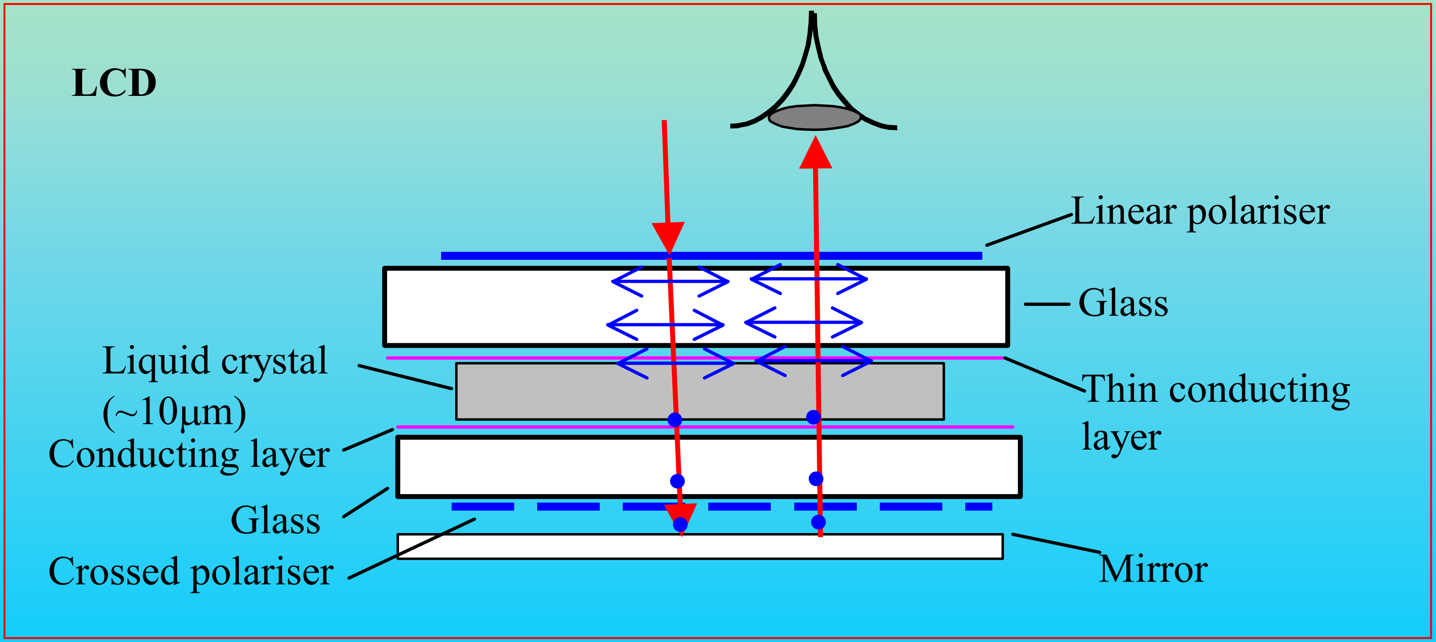
red - 0

yellow - C

blue - H bonds

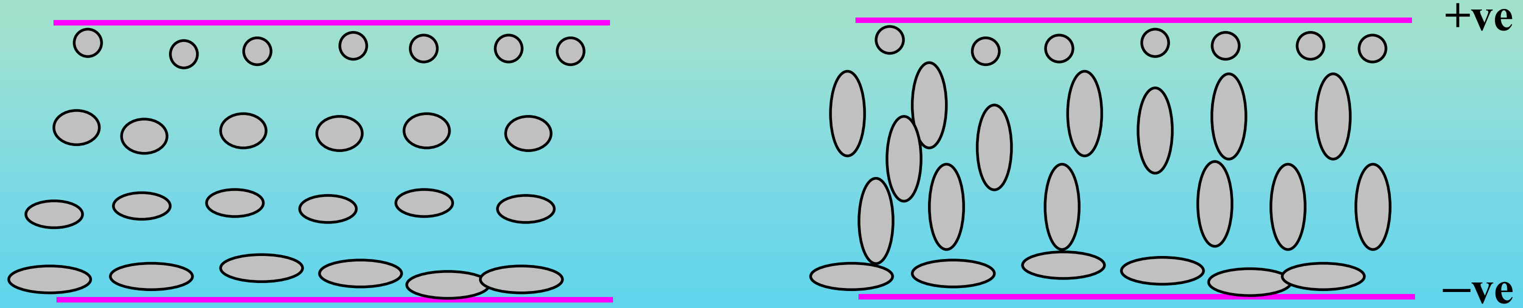
Liquid crystal displays

- An LCD pixel uses crossed polarisers to produce the dark state and an electrically induced change of polarisation to produce the bright state
- The popular **twisted nematic** LCD:



Molecular orientations with an LCD

- The alignment of molecules is induced by a surfactant to produce a highly optically active cell
- A small voltage is sufficient to re-align the molecules

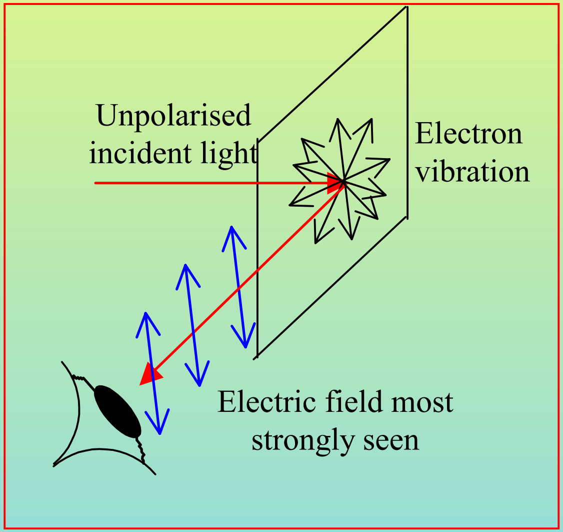


No voltage
Natural state with built-in 90° twist.

Applied voltage
Molecules away from surface re-align

Polarisation by scattering

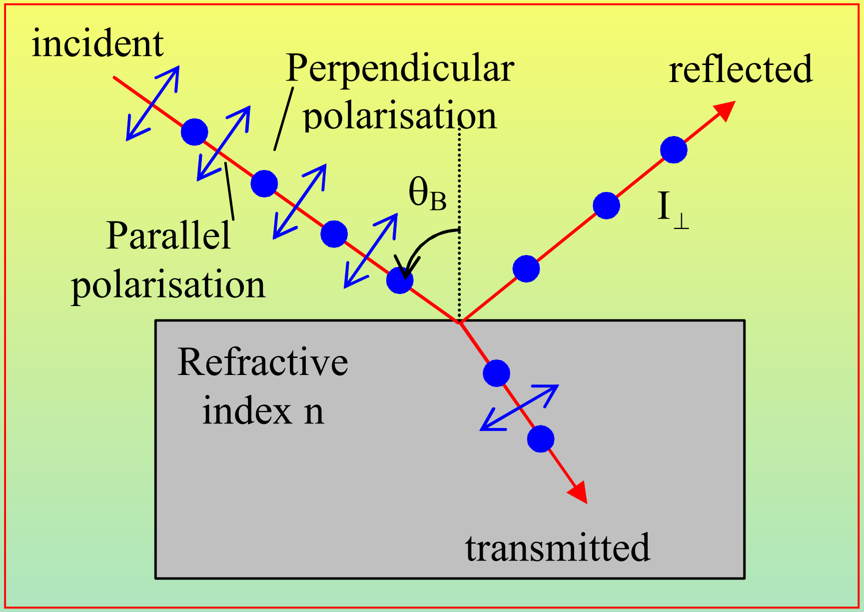
- Vibrating electrons emit light asymmetrically
  - most light is emitted ⊥ to their vibration direction
  - no light is emitted along their vibration direction



- Light scattered through 90° is strongly polarised
- The blue sky is polarised, particularly at 90° from the sun
  - use is made of this by insects, particularly bees, for navigating

The Brewster angle

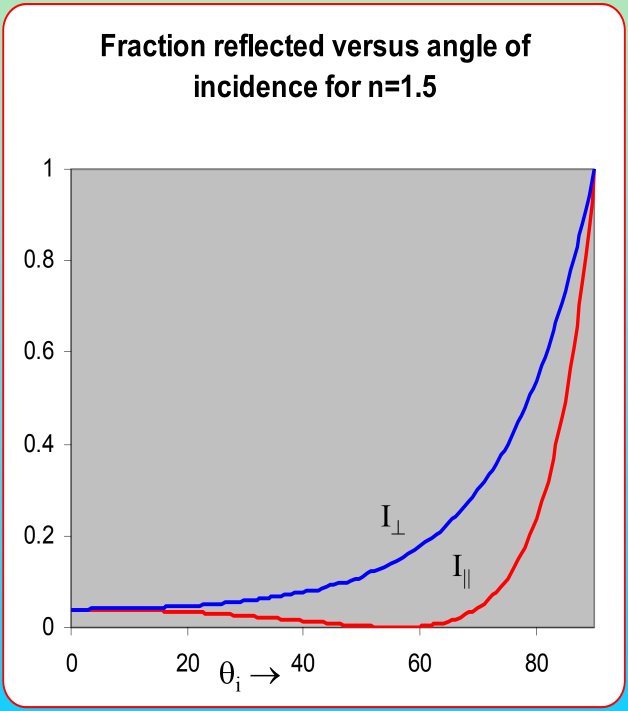
$$\boxed{\tan\theta_B = n}$$



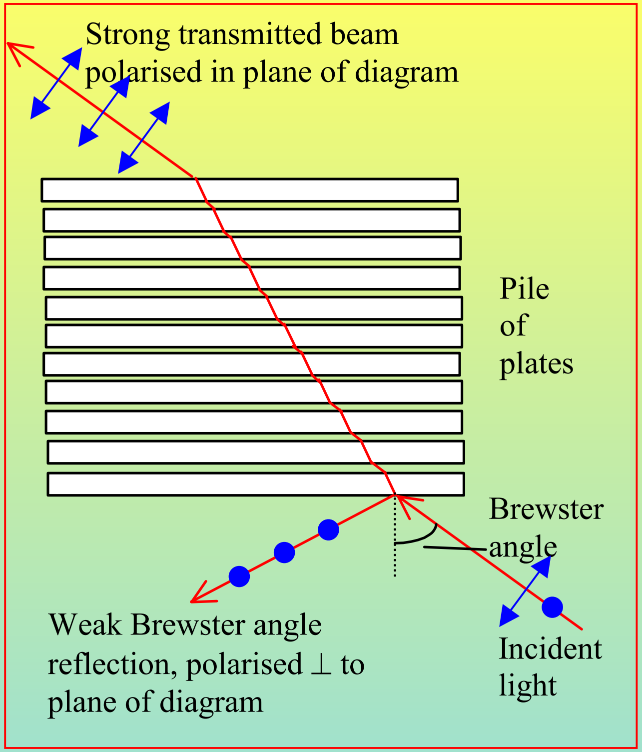
- The Brewster angle, θ_B, is the angle at which the reflected light is 100% polarised, ⊥ to the plane of incidence
- The reflected and transmitted rays are at 90°
- Example: for $n = 1.5$, $\theta_B = 56.3°$

Polarisation by reflection

- Fraction of light reflected at different angles of incidence depends on its linear pol’n



Fraction reflected versus angle of incidence for n=1.5



Strong transmitted beam polarised in plane of diagram

Pile of plates

Brewster angle

Weak Brewster angle reflection, polarised ⊥ to plane of diagram

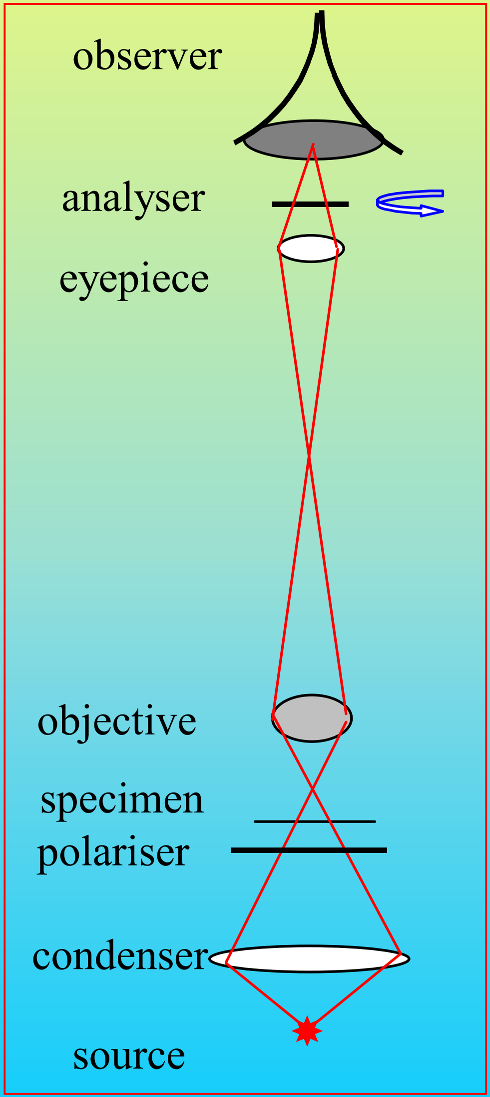
Incident light

- Observation in nature

- ‘Pile of plates’ polariser

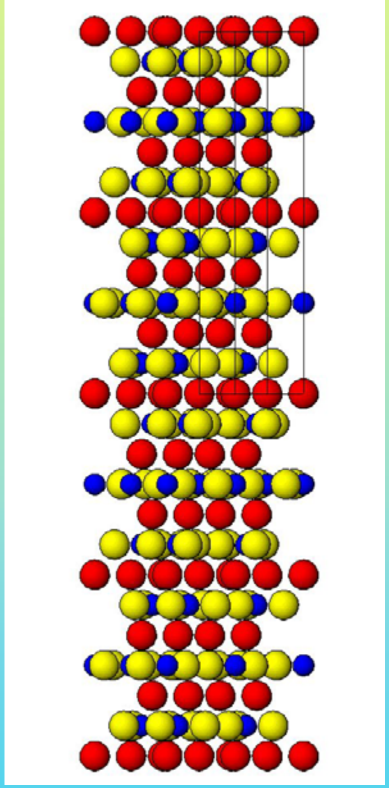
The polarising microscope

- The polarising microscope incorporates a 'polariser'
  - the sample is illuminated by linearly polarised light
- An 'analyser' allows the polarisation of the image to be investigated
  - the analyser is often set at 90° to the polariser
  - the geologists version is the **petrological microscope**



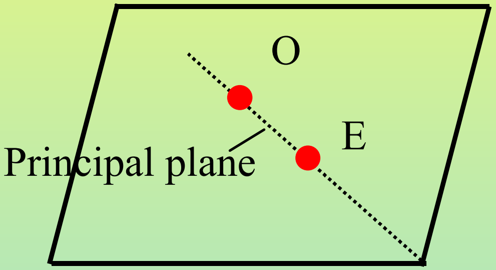
Birefringence

- Birefringence is a new range of phenomena opened up by the **anisotropy** of materials to the propagation of light
- These materials usually transmit light as **two** rays, even when one is incident
- $CaCO_3$ (calcite, Iceland spar) is the archetypical solid

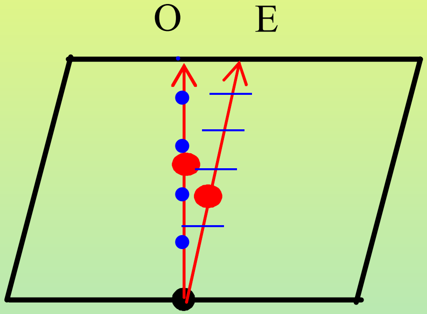


$CaCO_3$ viewed up hexagonal axis

Ordinary & extraordinary rays



Top view of rhomb



Side view of rhomb in principal plane

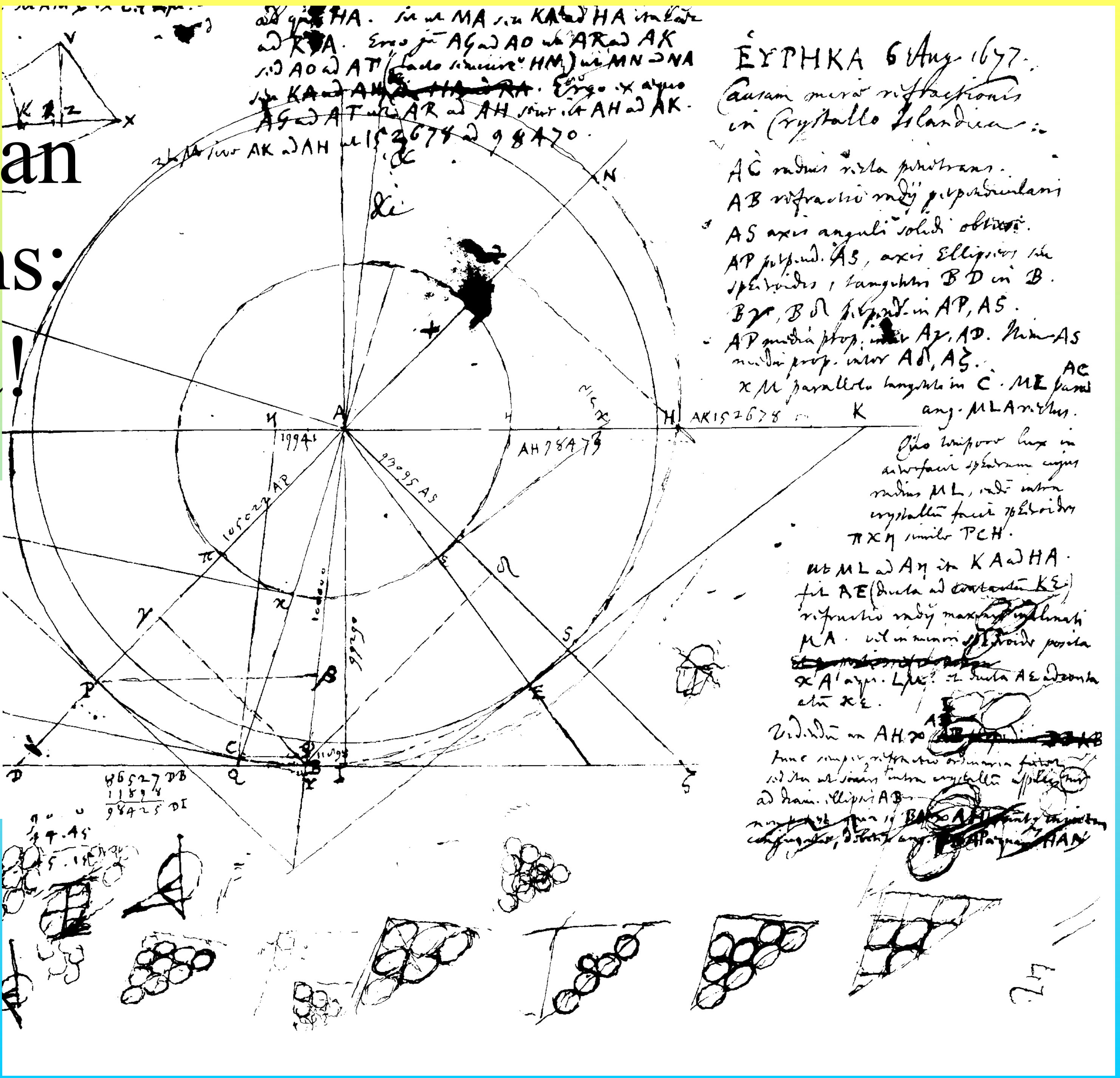
● indicates polarised ⊥ to principal plane

— indicates polarised ∥ to principal plane

● locates image of dot below surface

- The **ordinary ray** obeys Snell’s law
- The **extraordinary ray** deviates in a plane containing the optic axis direction of the crystal
  - such a plane is called a **principal plane**
- Both rays are linearly polarised at right angles to each other

Christiaan Huygens: euraka!



ΕΥΡΗΚΑ 6 Aug. 1677.

Causam miræ refractionis in Crystallo Islandica.

AC radius recta penetrans.

AB refractio radii perpendicularis

AS axis anguli solidi obtusi.

AP perpend. AS, axis Ellipseos sive sphæroidis, tangentis BD in B.

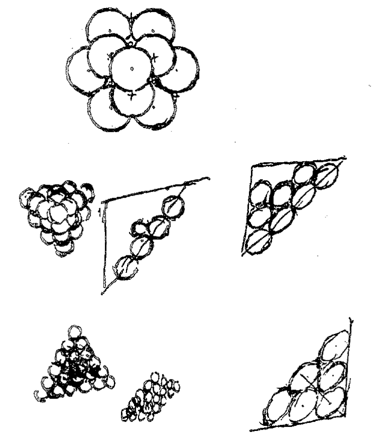
Bγ, Bδ perpend. in AP, AS.

AP media prop. inter Aγ, AD. Item AS media prop. inter Aδ, Aζ.

XM parallela tangenti in C. ME parall. AC ang. MLA rectus.

Quo tempore lux in aere facit sphæram cujus radius ML, eodem intra crystallum facit sphæroidem πXη similem PCH.

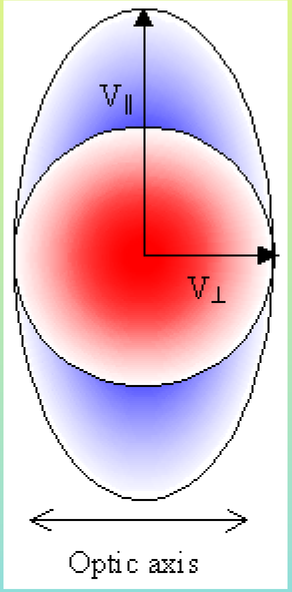
Ut ML ad Aη ita KA ad HA. fit AE (ducta ad contactum KE) refractio radii maxime inclinati MA. [illegible]

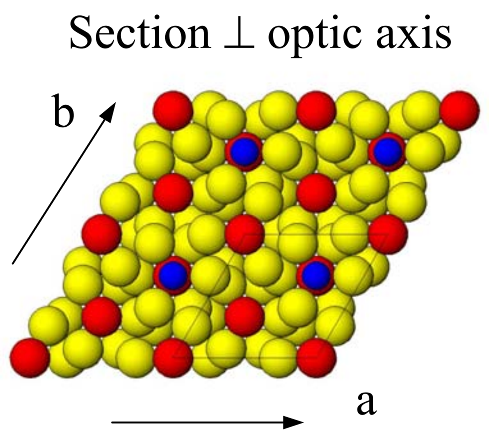


Huygens' packing drawings

Waves in a uniaxial crystal

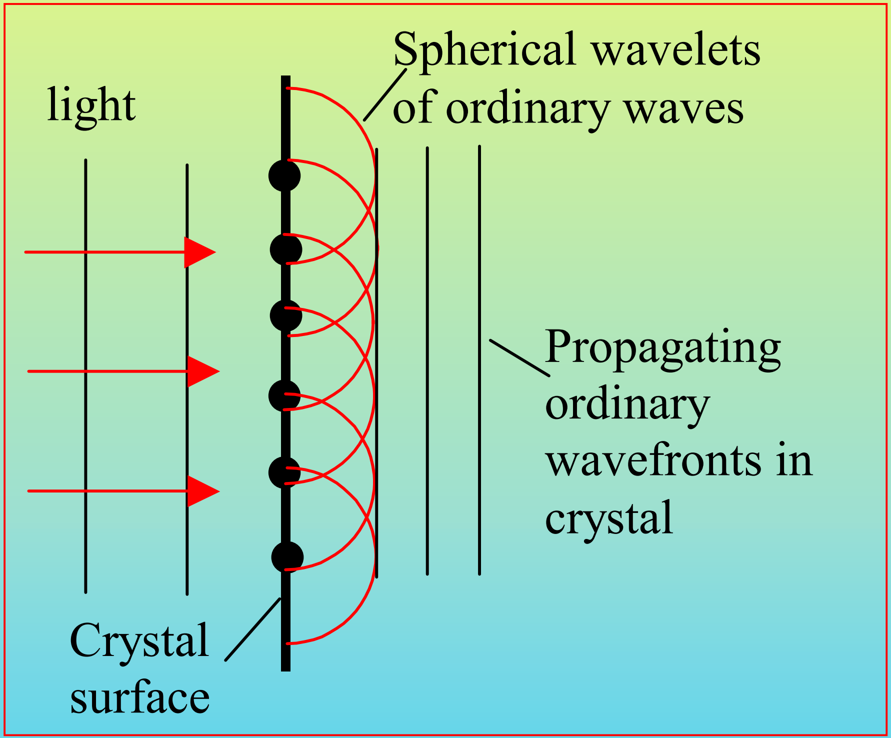
- Calcite **optic axis** || 3-fold axis
- Ordinary rays are propagated by an expanding spherical wave
  - the electric vector is ⊥ optic axis
  - refractive index $n_o = c/v_\perp$
- Extraordinary ray is propagated by an expanding ellipsoidal wave
  - the electric vector is || princ. plane
  - smallest refractive index $n_e = c/v_\parallel$



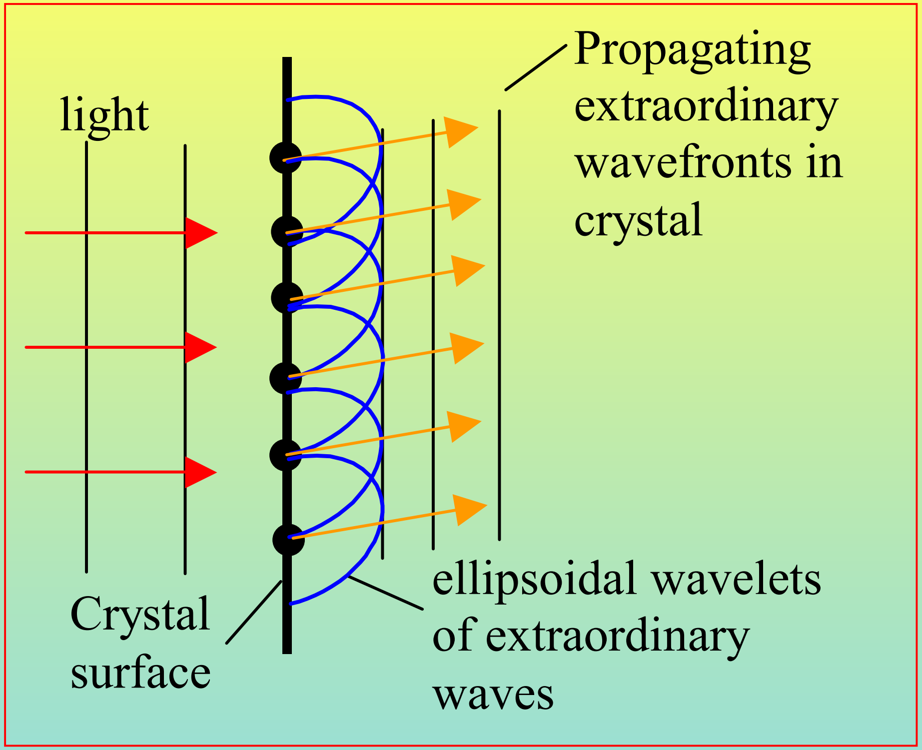


Propagating ordinary waves

- Ordinary waves propagate as you would expect from Huygens’ principle
- The refractive index n_o for calcite is 1.658
- n_e for calcite is 1.486
  - calcite is an example of a **negative uniaxial** crystal, because $n_e < n_o$



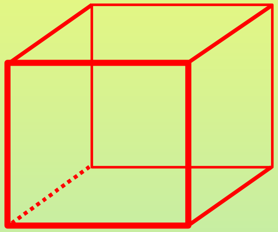
Propagation of extraordinary waves

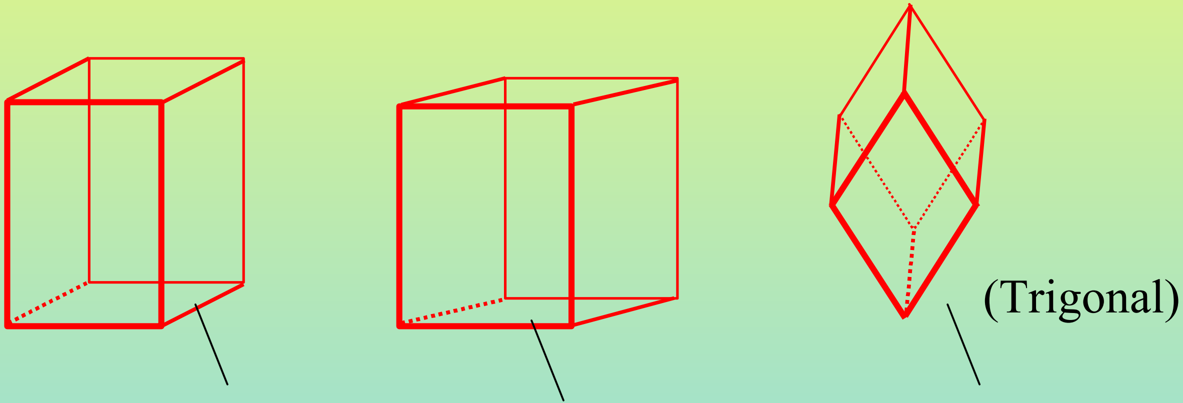


- Remember that extraordinary wavelets propagate as ellipsoidal wavefronts
- The axes of the ellipsoids are inclined to the surface
- The common tangent cuts the ellipsoids off to the side
- The direction of the propagating ray is therefore not perpendicular to the surface
  - inside an anisotropic crystal, the extraordinary light is generally not a purely transverse wave
  - **Biaxial** crystals have 2 extraordinary rays; they are complicated

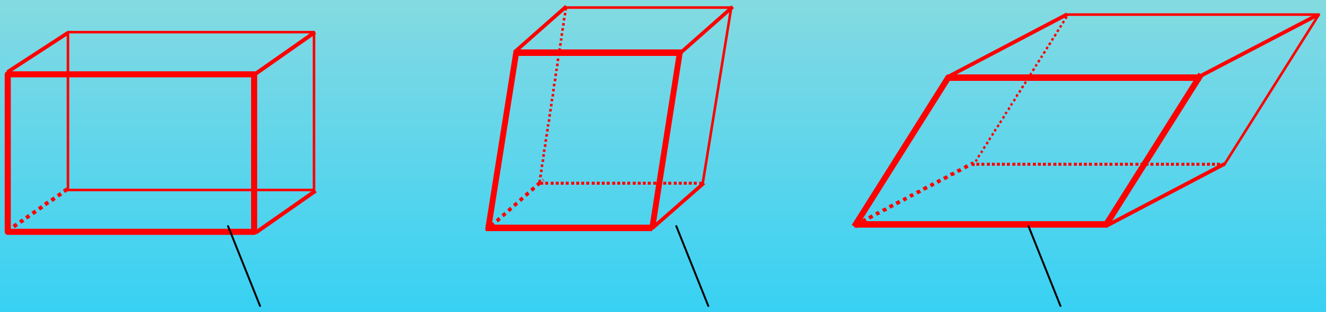
Birefringence is related to crystal class

- Cubic – **isotropic**





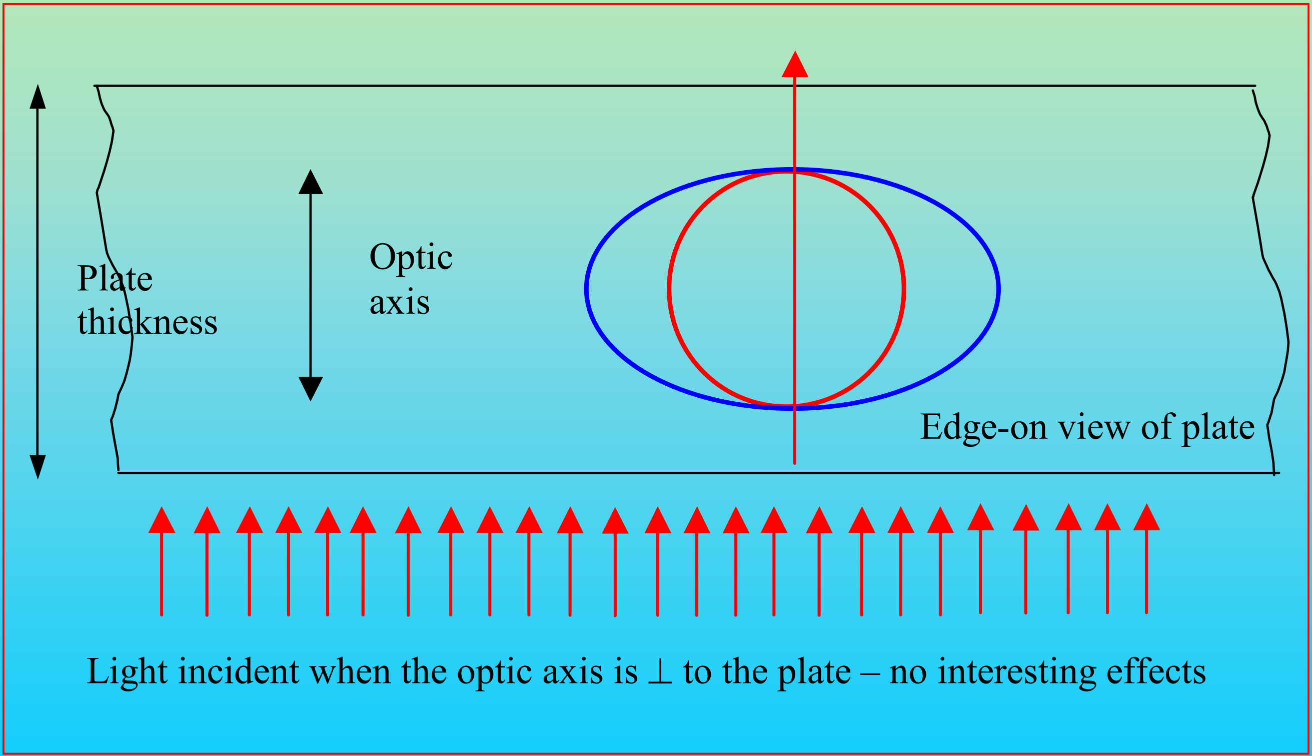
- Tetragonal, Hexagonal, Rhombohedral – **uniaxial**



- Orthorhombic, Monoclinic, Triclinic - **biaxial**

Light incident || optic axis

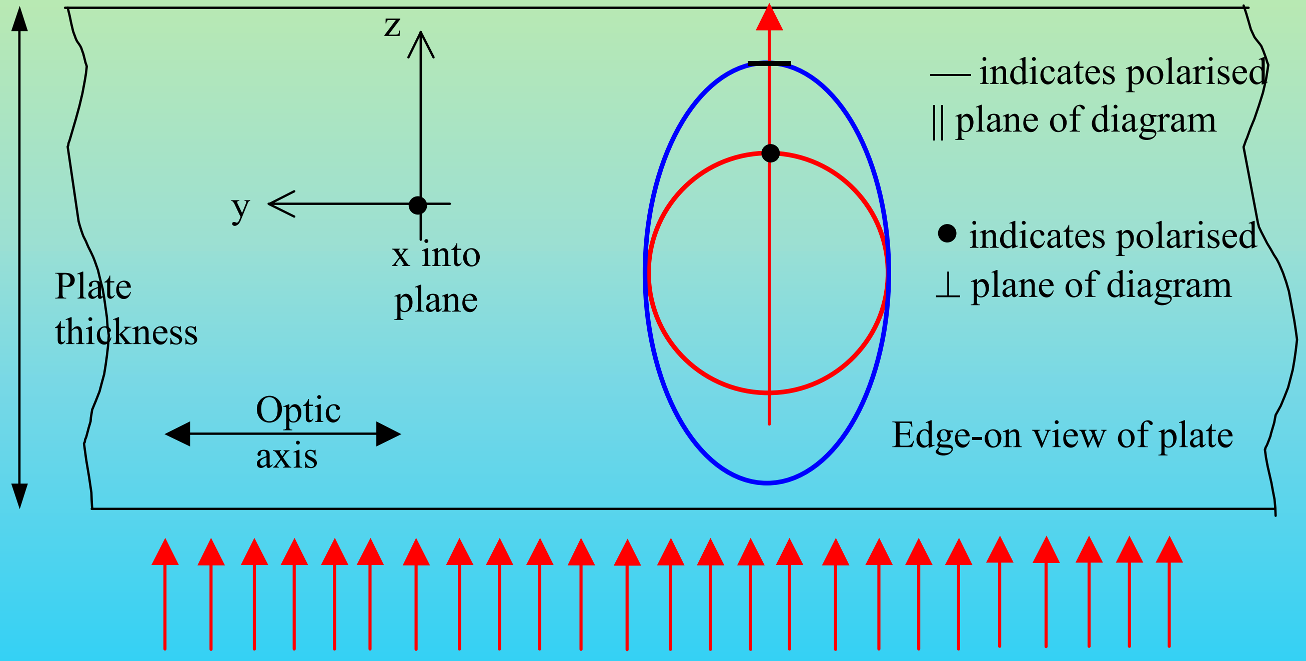
- Both rays travel together, producing no special effects



Light incident when the optic axis is ⊥ to the plate – no interesting effects

Light incident $\perp$ optic axis

- The 2 polarisations travel at speeds c/n_o and c/n_e, acquiring a phase difference



z

y

x into plane

Plate thickness

Optic axis

— indicates polarised || plane of diagram

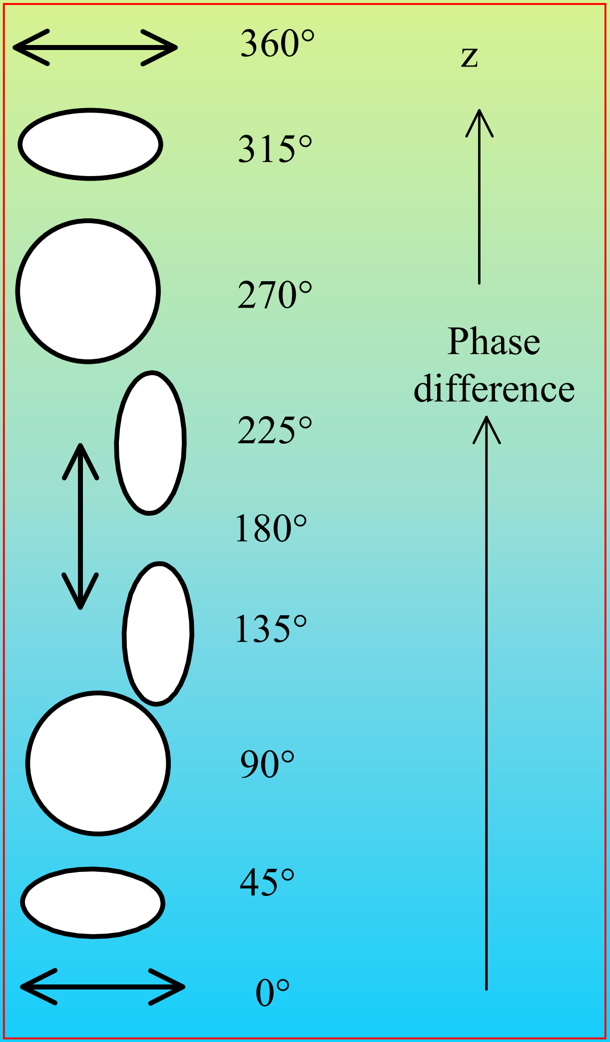
● indicates polarised $\perp$ plane of diagram

Edge-on view of plate

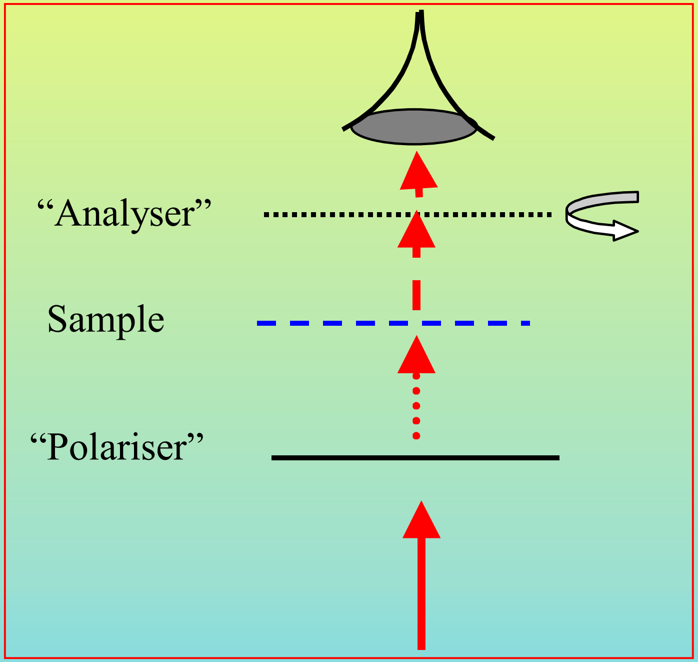
Light incident when the optic axis is || to the plate

Polarisation change during propagation

- The phase change between the 2 rays is $z(n_o - n_e)2\pi/\lambda_{vac}$
- If the 2 rays start off with equal amplitude, then the diagram shows how the polarisation changes with z, the distance travelled
  - the sequence happens every 3 µm in calcite
  - 100 µm is more typical of minerals



Minerals and the microscope



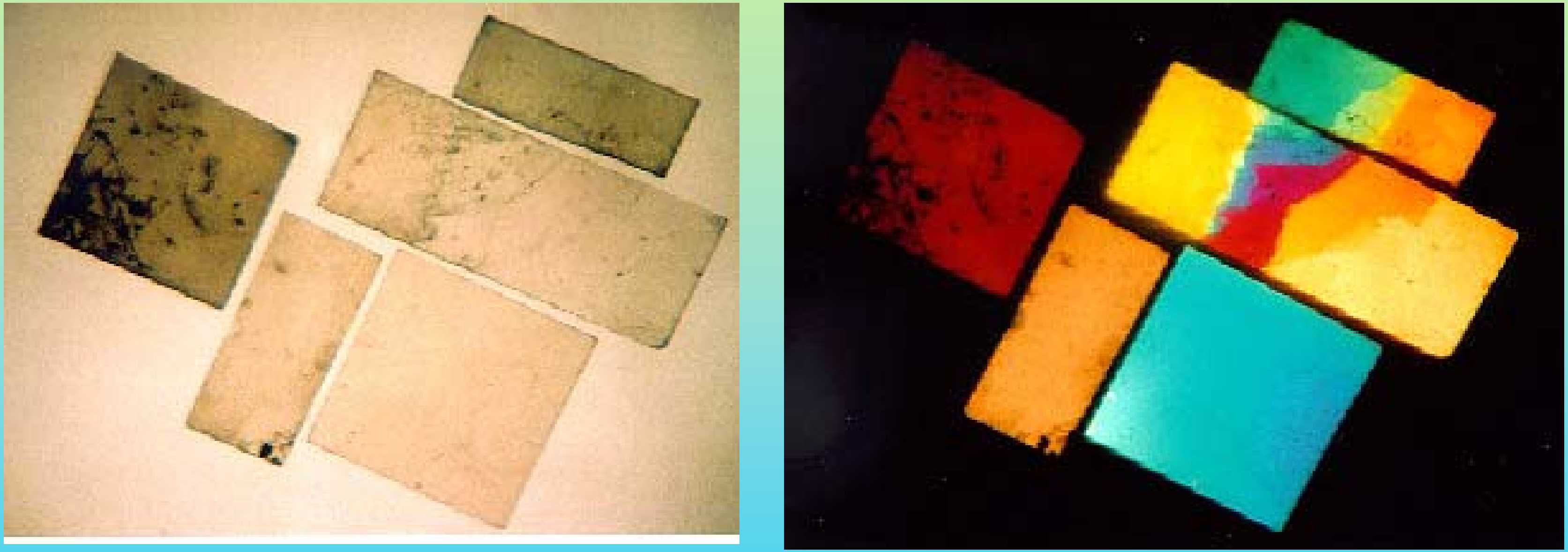
Appearance of Moon rock in the polarising microscope

picture courtesy :
micro.magnet.fsu.edu

- Anisotropic material appears black; birefringent material appears with **polarisation colours**
  - the most intense colours are when the optic axis is at 45°
  - **extinction** occurs when the optic axis is || or ⊥ to the polariser
  - additional colouring is provided by **pleochroism**, selective polarisation dependent absorption of some colours

Demonstration example

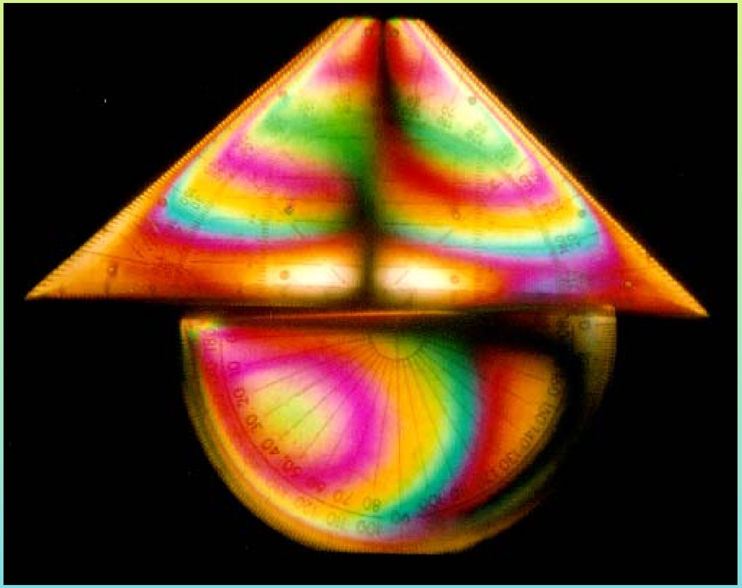
- The first picture shows several sheets of mica of different thicknesses seen in ordinary light

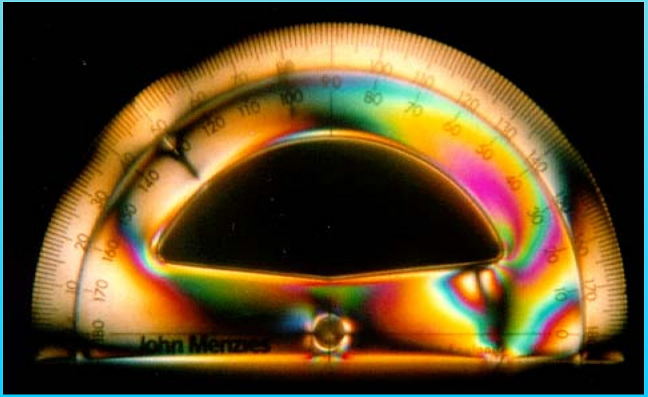


- The second picture, the same sheets between crossed polaroids

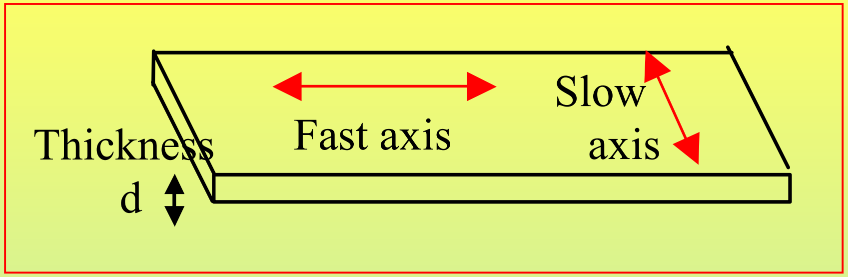
Strain in transparent materials

- Colours are caused by strain induced birefringence
  - also by variations of thickness
  - for a 1 mm thick material, 360° phase shift is caused when $(n_o - n_e) \approx 5\times10^{-4}$





Retarders

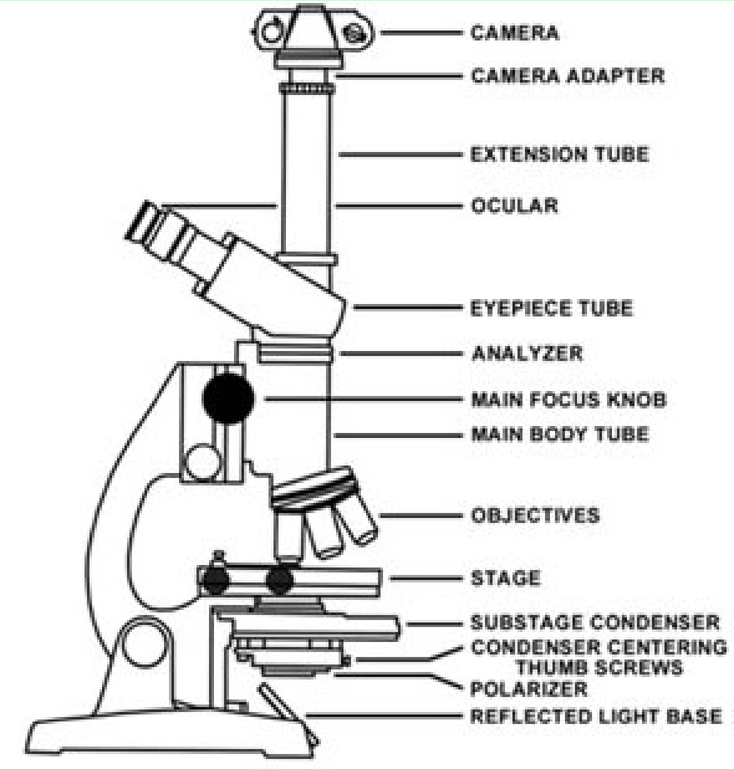


- A **retarder** is a uniform plate of birefringent material whose optic axis lies in the plane of the plate. Retarders can be used to
  - make circularly polarised light
  - analyse elliptically polarised light
  - interpret colours in the polarising microscope
- **Slow axis** is optic axis for calcite
  - **fast axis** is ⊥ slow axis
- Phase retardation Δϕ, in radians

$$\boxed{\Delta\varphi = k_{vac} d \left(n_{slow} - n_{fast} \right)}$$

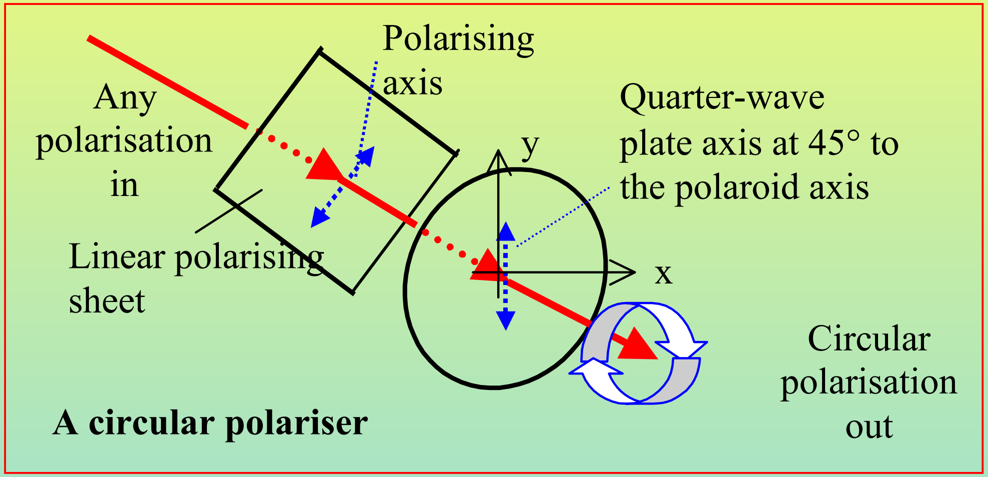
Retardance

- A **full-wave plate** retards the slow wave relative to the fast wave by 2π radians
- A **quarter-wave plate** retards by $\pi/2$
  - in terms of phase, the retardance is **chromatic**
  - the **retardance** may be measured in wavelength
    - e.g. a retardance of 250 nm, which is $d(n_{slow} - n_{fast})$
- Why bother?
  - e.g. in the polarising microscope, sliding in a retarding plate between sample and analyser enables a microscopist to decide how birefringent the sample is, helping identification of the sample



Making circularly polarised light

- Circular polarisation is made by shining linearly polarised light at 45° onto a quarter-wave retarder



- The output looks like:

$$
\boxed{\begin{aligned} E_x &= E_o \cos(kz - \omega t) \\ E_y &= \pm E_o \sin(kz - \omega t) \end{aligned}}
$$

  - the + sign occurs if the slow axis is || y direction, giving right circularly polarised output
    - -ve sign for slow axis ⊥ to y axis, giving left circularly polarised light