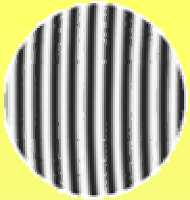
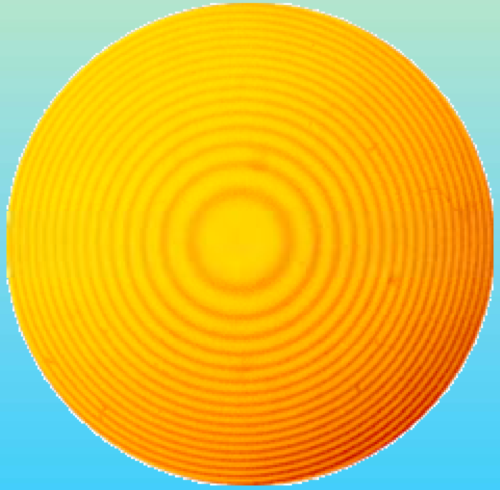
Interference of light



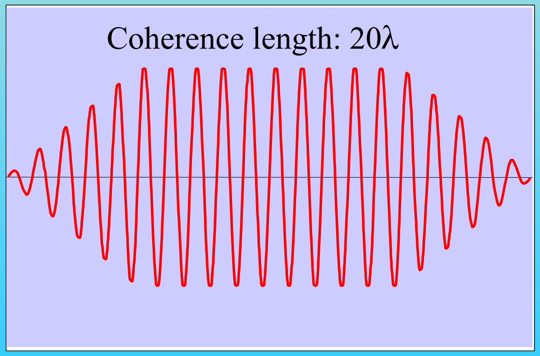
- Interference is fascinating, useful and subtle
- First discovered by Thomas Young
  - explains ‘interference colours’ seen in the natural world
  - has spawned the subject of **interferometry**, a variety of techniques for precision measurement
  - raises deep questions about the fundamental nature of light



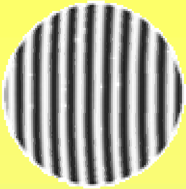
Ordinary illumination

- Light waves are too quick for detectors to record the electric field
  - remember: $I \propto \langle E^2 \rangle$
- Light waves are very short lived
  - each light packet acts independently
  - the total illumination (the irradiance) is the sum of the irradiance produced by each contributing source
  - symbolically:

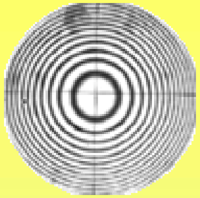
$$I_{total} = I_1 + I_2 + I_3 + I_4 + \ldots..$$



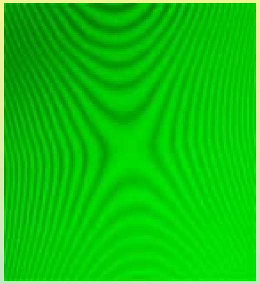
Temporal coherence: 20 periods



Interference fringes



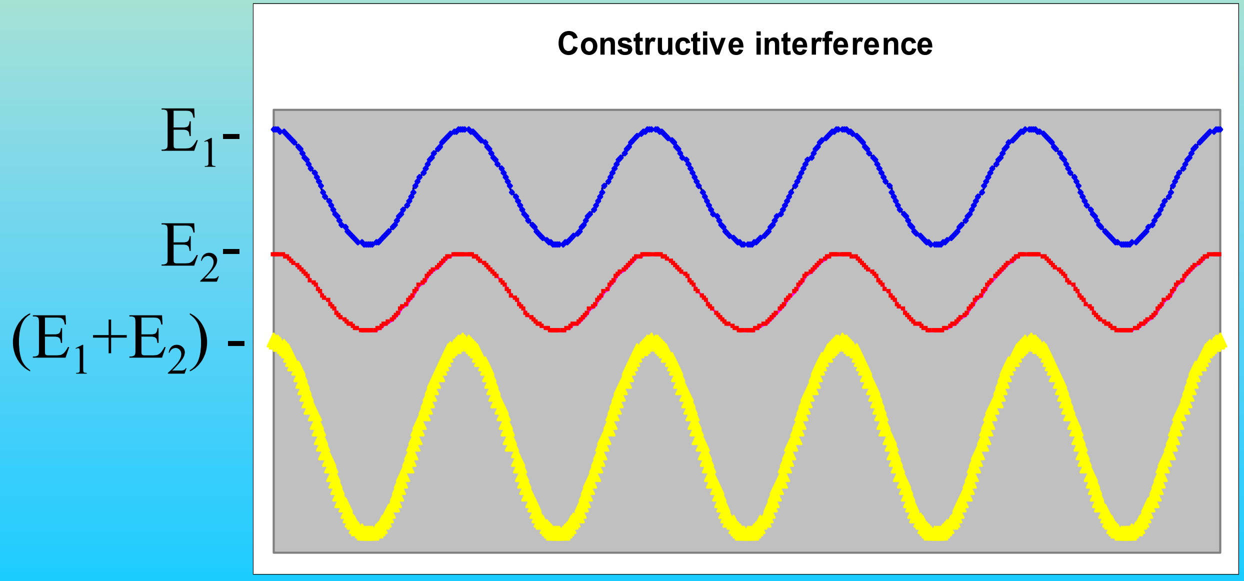
- Interference fringes are a series of bright and dark bands
  - sometimes straight, sometimes circular, sometimes more complicated



- When light waves interfere, you add the waves together first, **then** find the irradiance
  - e.g. for 2 waves: $I = \langle (E_1 + E_2)^2 \rangle$
- The limits of what can happen are called **constructive interference** *and* **destructive interference**

Constructive interference

- The two waves are exactly **in phase**
  - in the example shown, the blue wave (E_1) has amplitude 3 units and the red wave (E_2) has amplitude 2 units
  - the constructive interference has amplitude 5 units



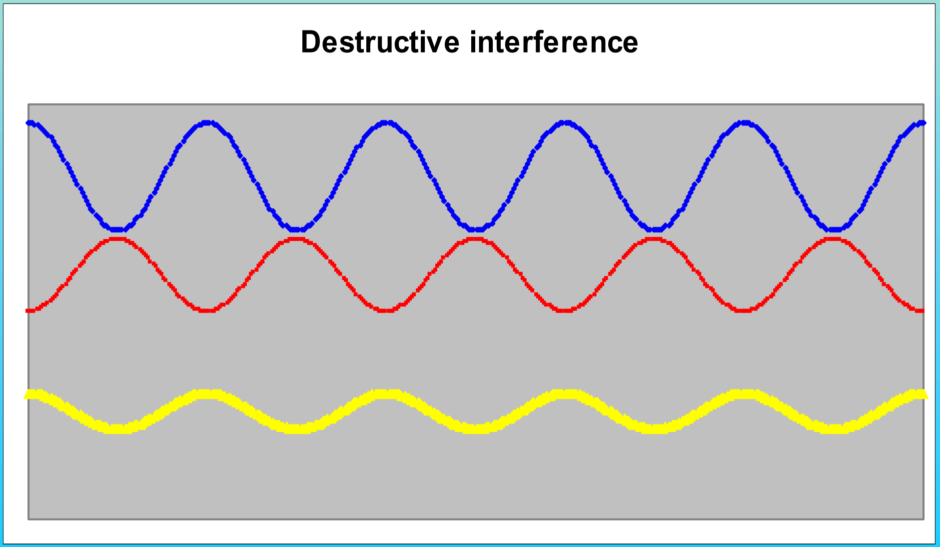
Destructive interference

- The two waves are exactly **out-of-phase**
  - in the example shown, the blue wave (E_1) has amplitude 3 units and the red wave (E_2) has amplitude 2 units
  - the destructive interference has amplitude 1 unit

E_1 -

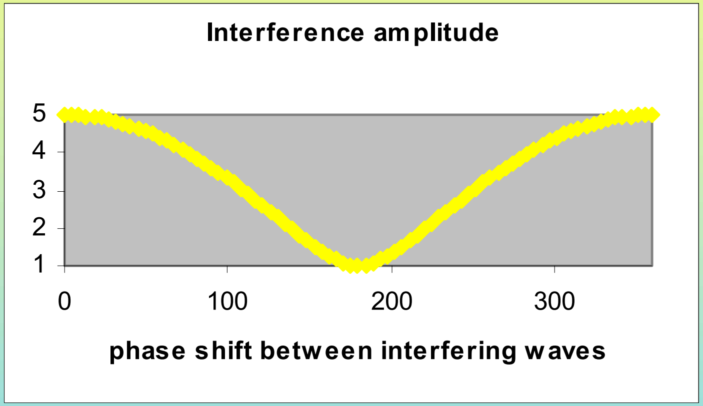
E_2 -

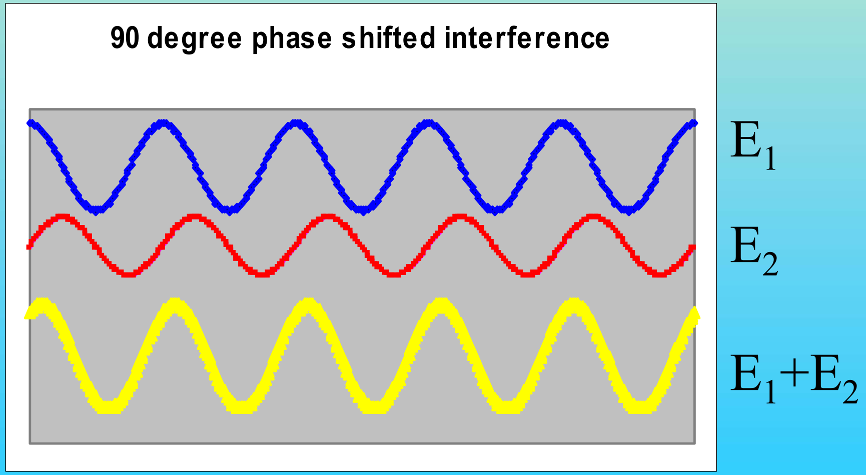
(E_1+E_2) -

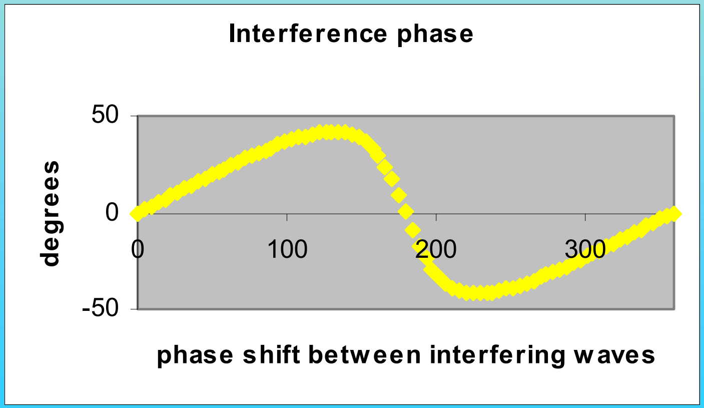


Intermediate phase interference

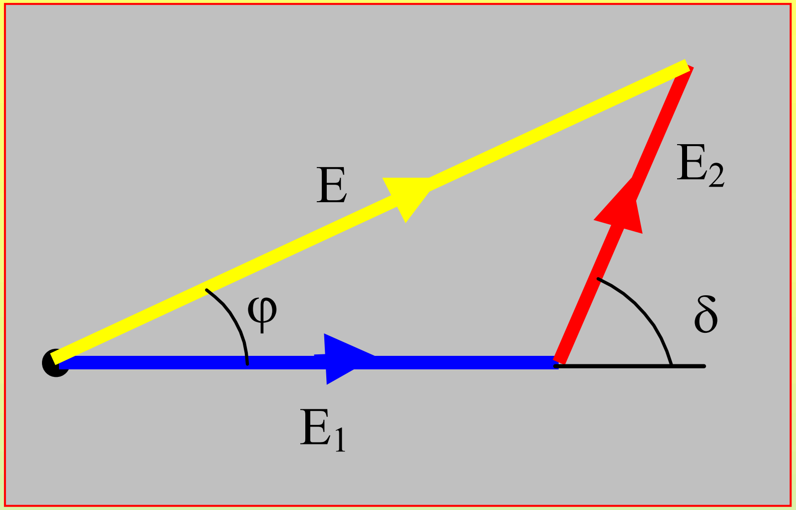
- The sum of two cosine waves is always a cosine wave
  - the amplitude lies between the extremes of constructive and destructive interference







Phasors

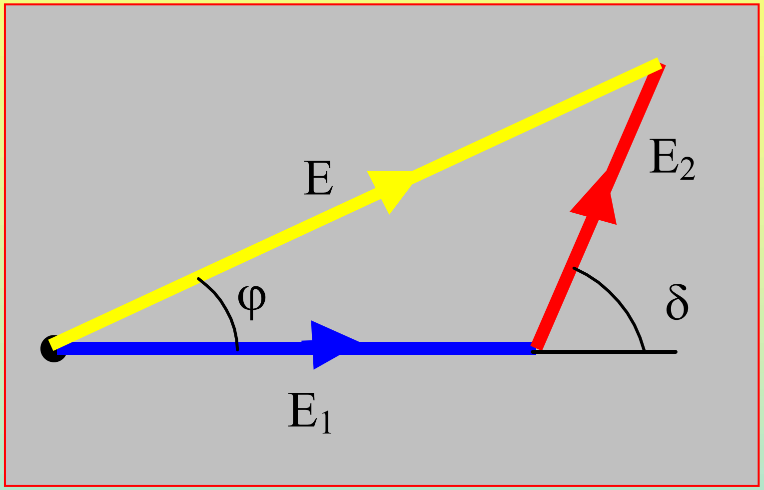


- Phasors are a diagrammatic help for adding waves
- Each wave is represented by a line whose length represents the amplitude and whose angle from the x-axis represents the phase
- Add the phasors end-to-end to find the amplitude and phase of the sum of the waves
- The diagram shows the addition of our two waves with a phase angle of about 60°

Mathematically speaking

- Applying the cosine rule to the phasor triangle gives:

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2\cos(\delta)$$



- In terms of irradiance:

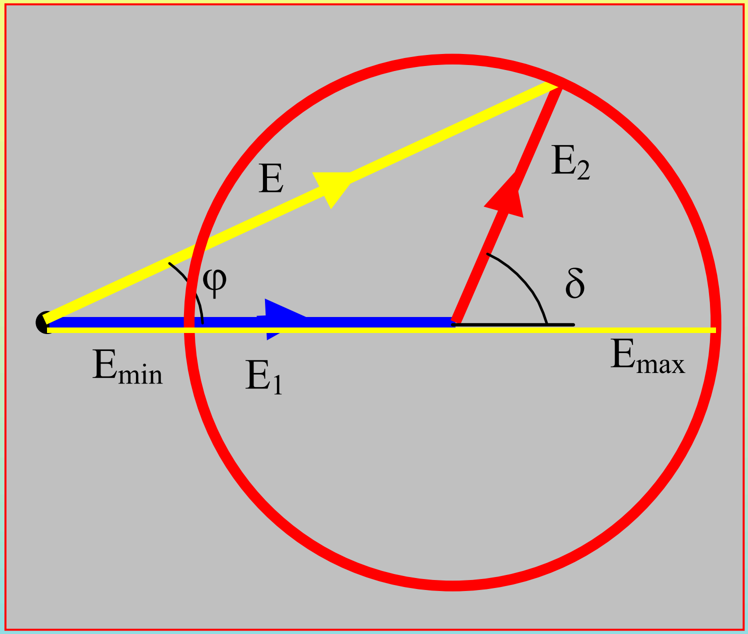
$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\delta)$$

- If the two waves have equal irradiances, $I_1 = I_2 = I_o$, say, then:

$$\begin{aligned} I &= 2I_o + 2I_o\cos\delta \\ &= 2I_o(1+\cos\delta) \\ &= 4I_o\cos^2(\delta/2) \end{aligned}$$

All possible phases of E_2

- All possible phases of E_2 are represented by the end of E_2 lying around a circle
- It is easy to see that the maximum value of the amplitude will be when the two waves are in phase, the minimum when the two are exactly out-of-phase

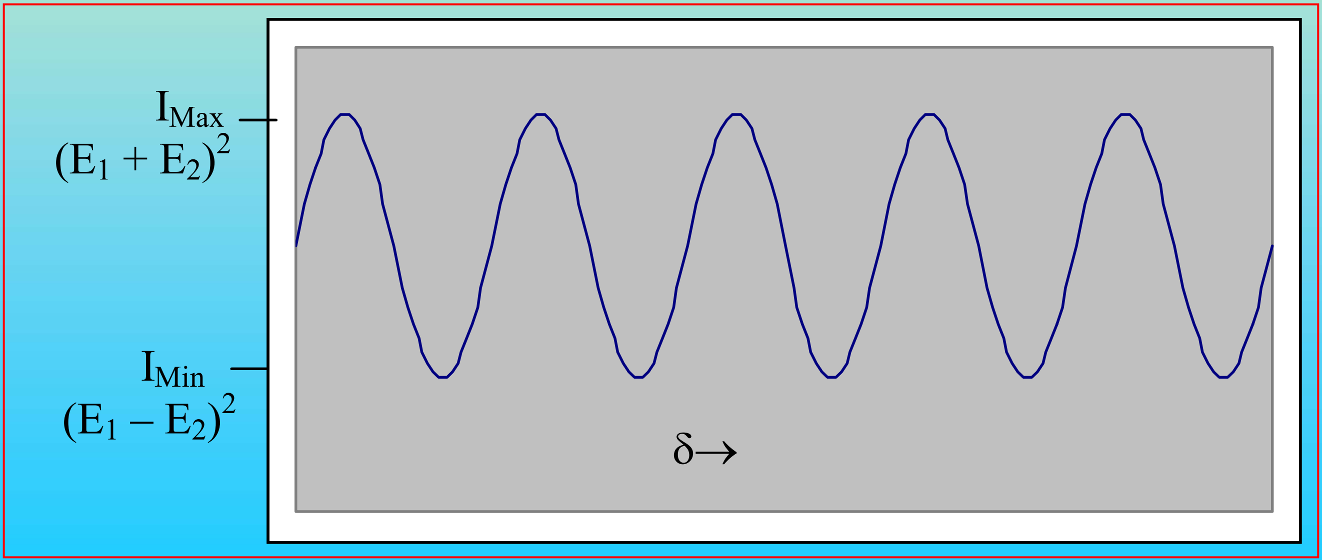


- The phasor diagram gives the right answer for all intermediate cases

Fringe visibility

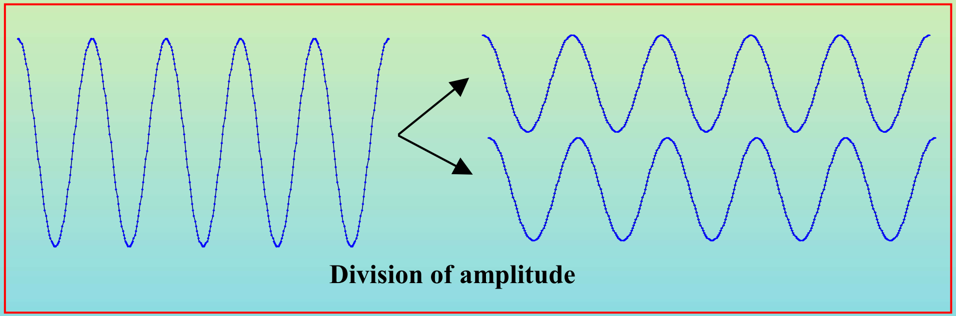
- The visibility of fringes decreases as the minimum gets stronger
- A simple measure of percentage visibility:

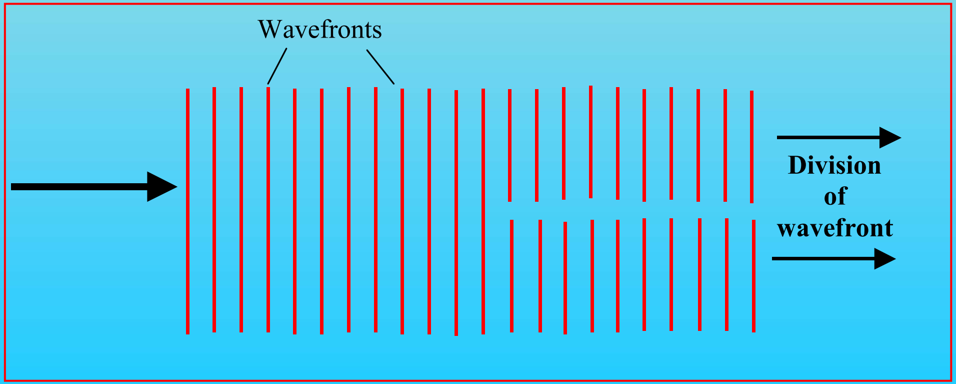
$$V = \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100\%$$



Waves interfere with themselves

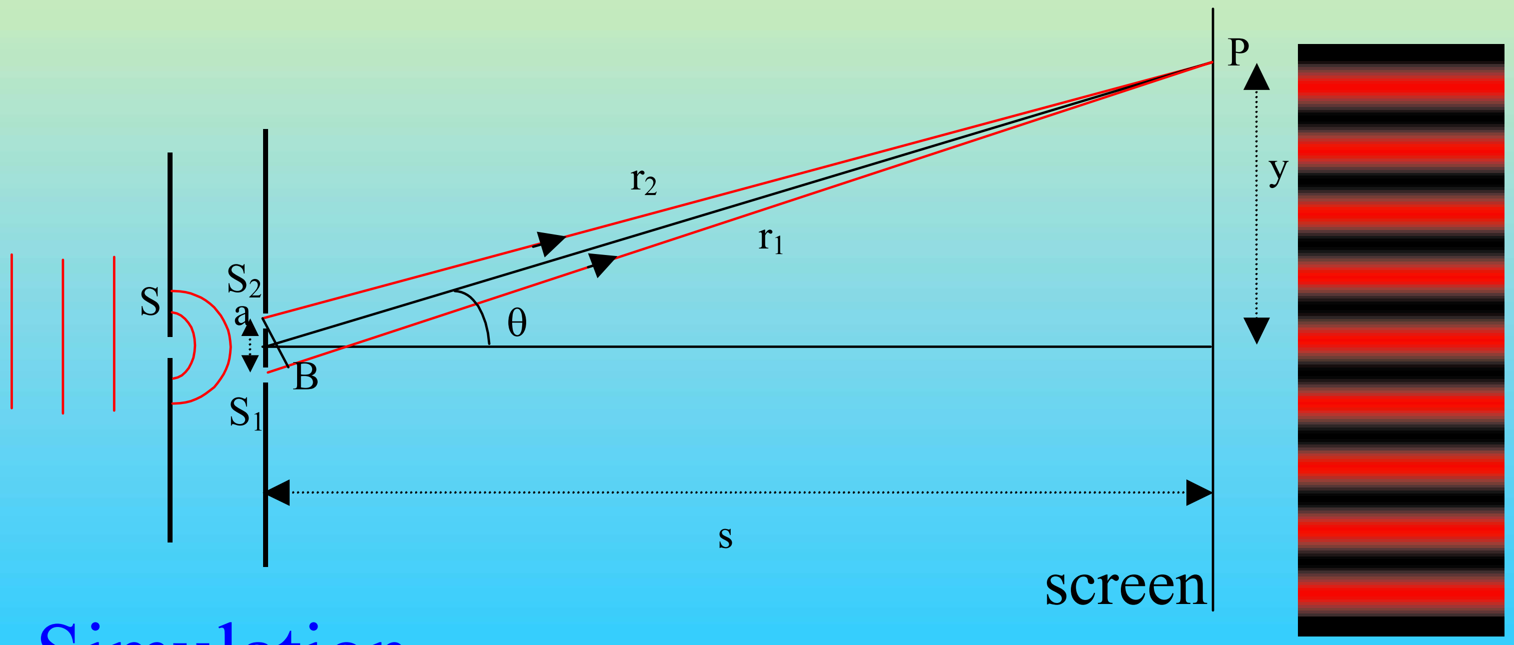
- Interfering waves must stay in step
  - they have to be **coherent**
  - they must be **monochromatic** – of one wavelength
- Interference is obtained by arranging that part of any wave interferes with itself
  - **division of amplitude**
  - **division of wavefront**





Young's slits interference

- Young's slit experiment is one of the world's great experiments
- The slits S_1 and S_2 act to divide the wavefront



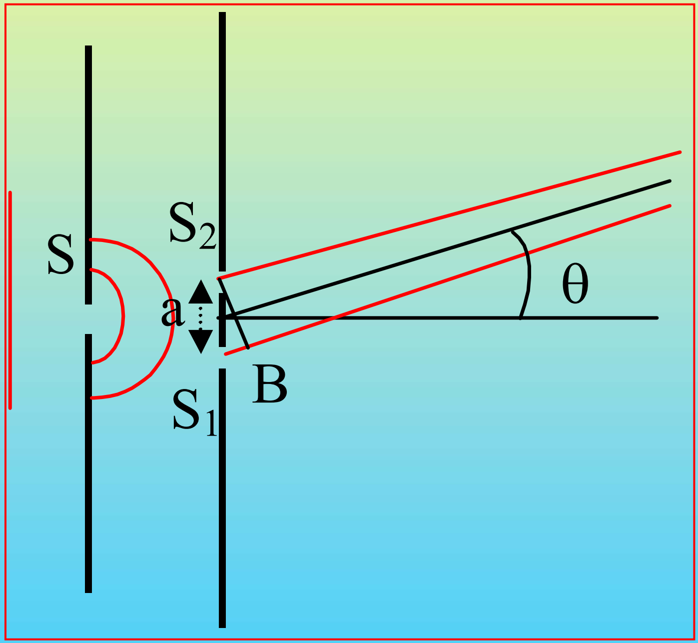
- Simulation

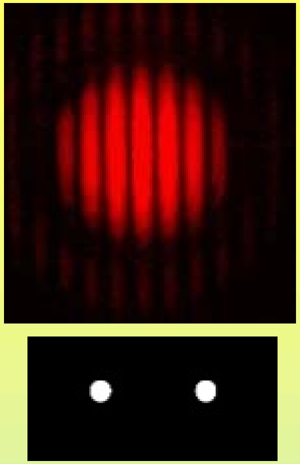
Location of Young's fringes

- Look closely at the path difference near the slits
  - constructive interference when $m\lambda$ = extra path length from S_1 = $S_1B = a \sin\theta \cong a\theta$
  - hence the m[th] bright line at $\theta_m = m\lambda/a$
  - equivalently, distance up screen $y_m = s\theta_m = m\lambda s/a$
  - spacing between neighbouring fringes $\Delta y = \lambda s/a$
  - $\cos^2$ fringes with irradiance:

$$I \propto \cos^2(kay/2s)$$

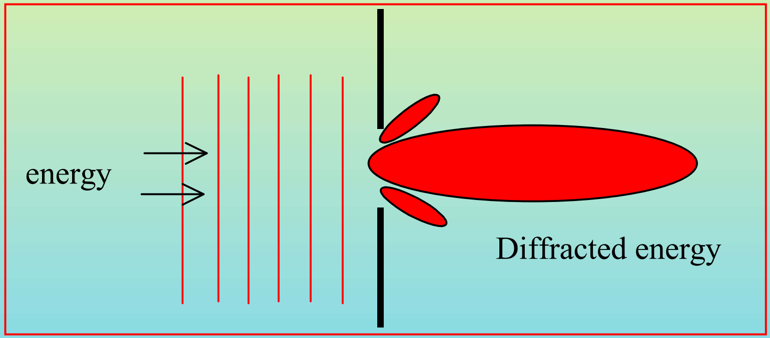
  - e.g. a = 0.2 mm, s = 2 m; λ = 550 nm, gives Δy = 5.5 mm





Rôle of diffraction

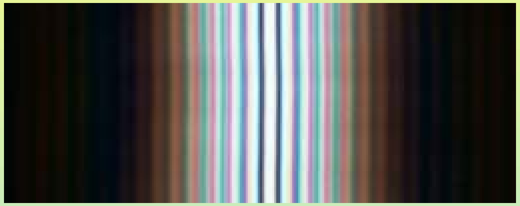
- Diffraction is the spreading out of light in directions not predicted by ‘straight line propagation’
  - remember this diagram from earlier:



- Diffraction is essential for Young’s slits to work, for it provides the illumination of S_1 and S_2 by S, and the light at angle θ away from the straight-through position after the two slits

Deductions from Young's experiment

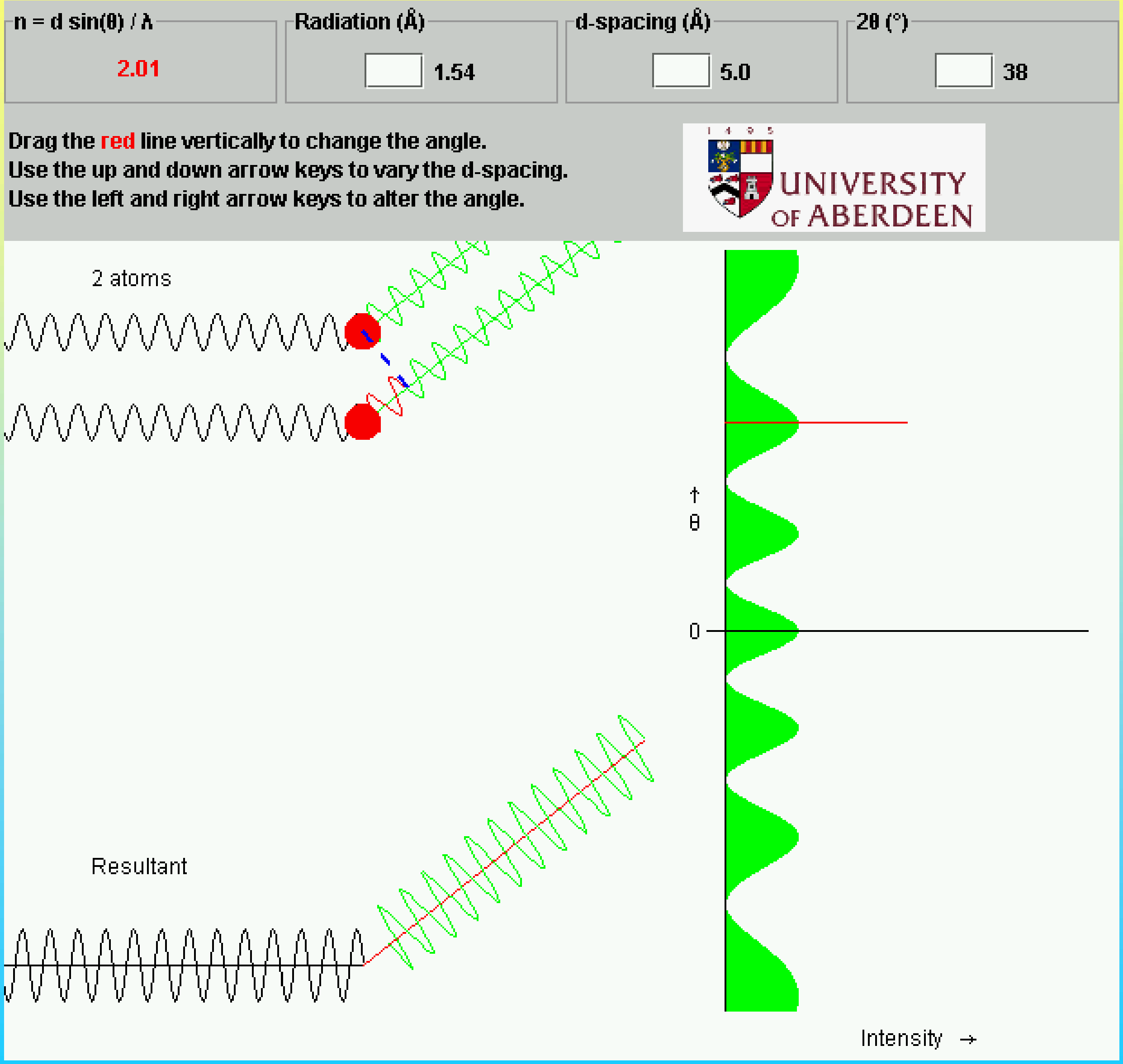
- By measuring the distance between neighbouring fringes, the wavelength of light can be deduced, even though it is very small
- Even with white light, a few coloured fringes can be seen around the central white fringe, before the colours wash out



- By putting a wedge of material across S_1 the path length can be increased until the fringes disappear, giving a measure of the coherence of the light source
- S can be disposed of if we use a laser, which has *transverse coherence* across its beam
- What happens when the intensity of the light is so low that only single photons pass through the apparatus at a time?
- The equivalent of Young's slits work for electrons, neutrons and other particles with de Broglie wavelength $\lambda = h/p$

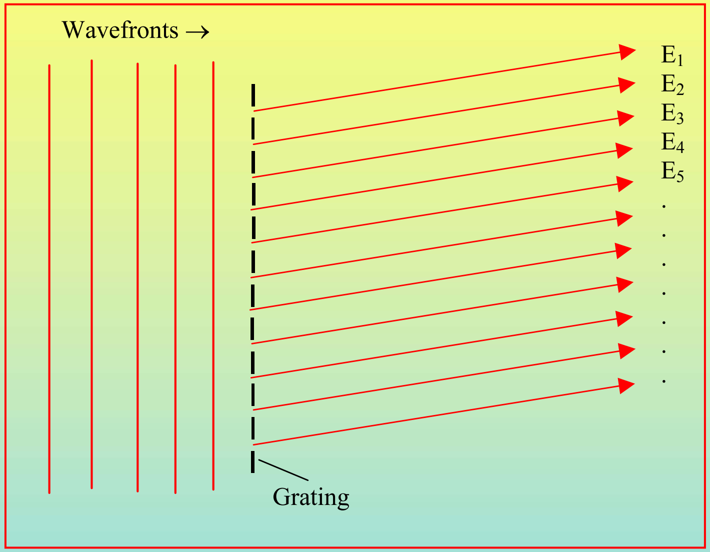
Interference applet - 1

- On our web pages
  - red dots can diffract at a chosen angle
- observe extra path difference
- observe intensity changes with angle and dot separation



Diffraction gratings

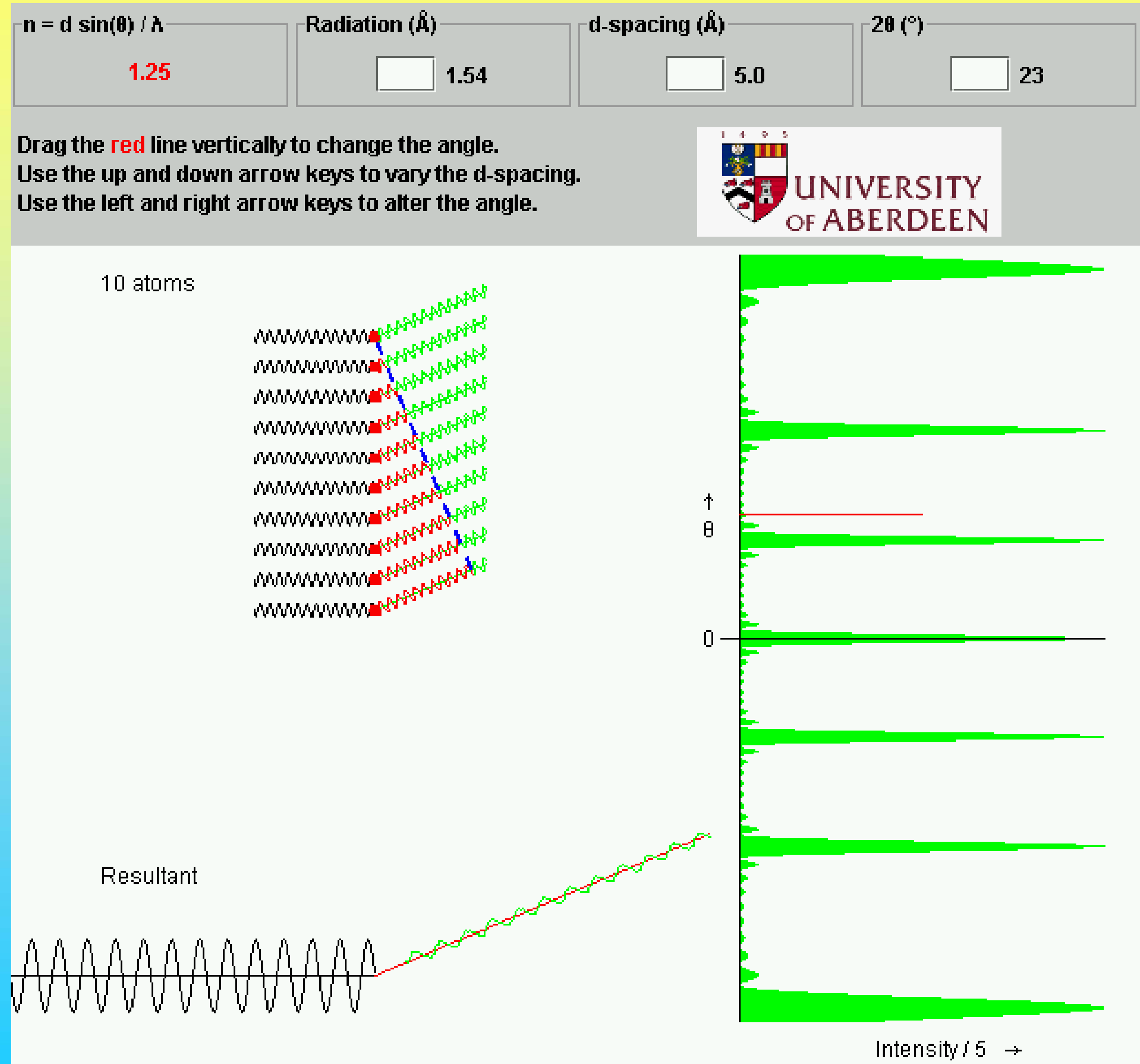
- A diffraction grating is a widely used central element in spectrometers
  - gratings spread out the light into its spectrum, usually much better than prisms



- Diffraction gratings consist effectively of a great many slits, perhaps between 10^4 and 10^5
- Diffraction gratings work by *interference*, the theory being only a simple extension of Young’s slit ideas

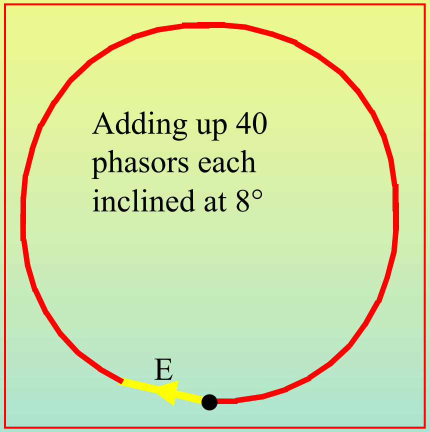
Interference applet - 2

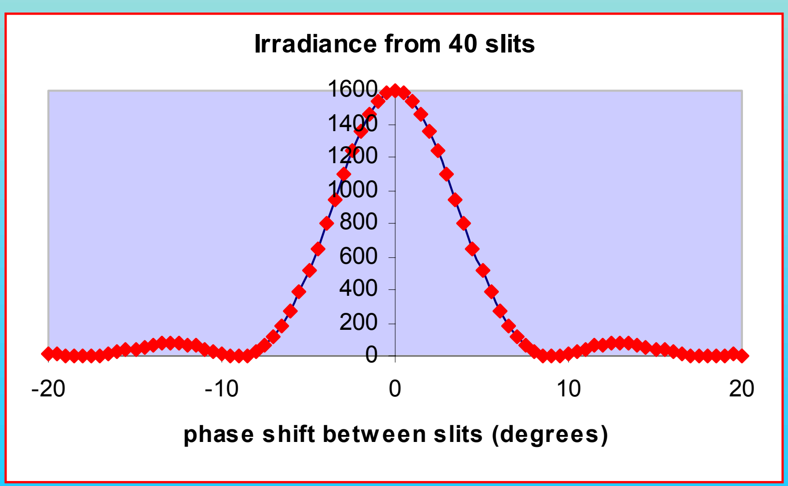
- Variant with 10 sources
- note build-up of path difference
- Note sharp peaks



Explanation with phasors

- Consider 40 slits. If the phase difference between neighbouring slits is 0° or 360°, then the total intensity is given by 40 phasor lines, end-to-end
- If the phase difference is only 8° different, then the phasors curl around giving a small total

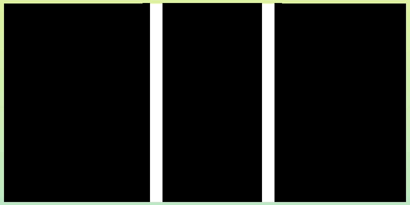




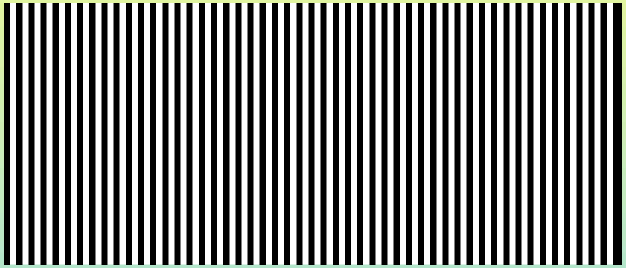
- The calculation alongside shows that a phase difference of 4° will reduce the irradiance to a half; 9° will reduce the irradiance to zero

Comparison between 2 and 50 slits

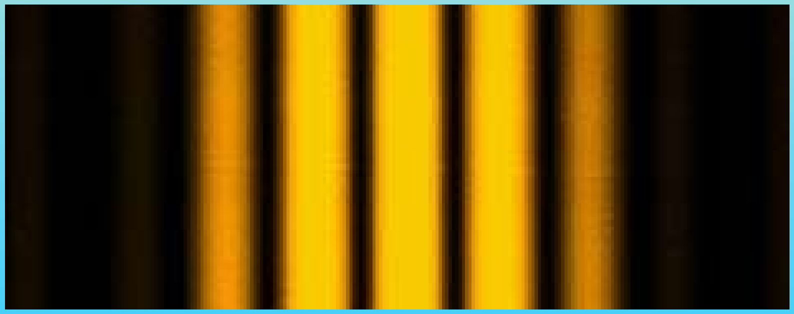
- 2 slits



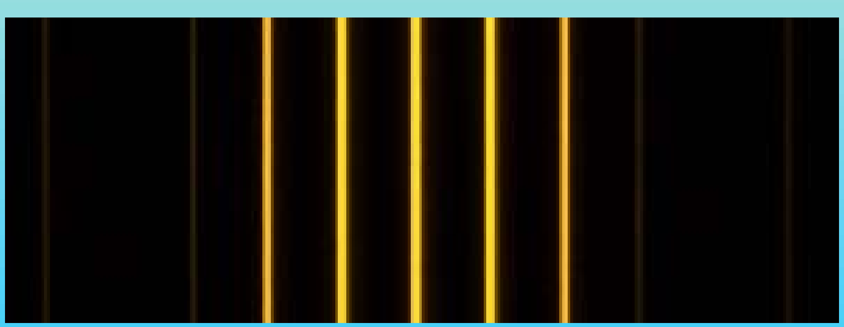
- 50 slits



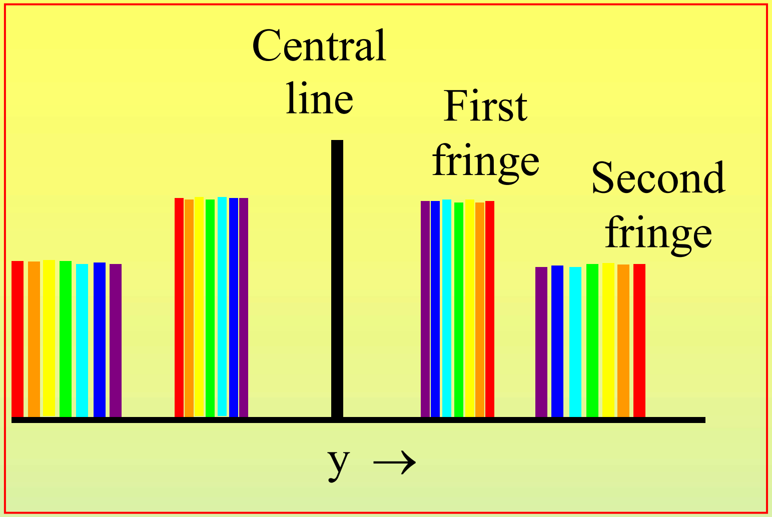
- Interference pattern



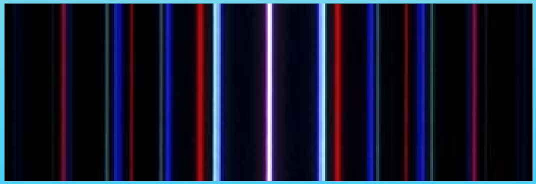
- Interference pattern



Formation of spectra

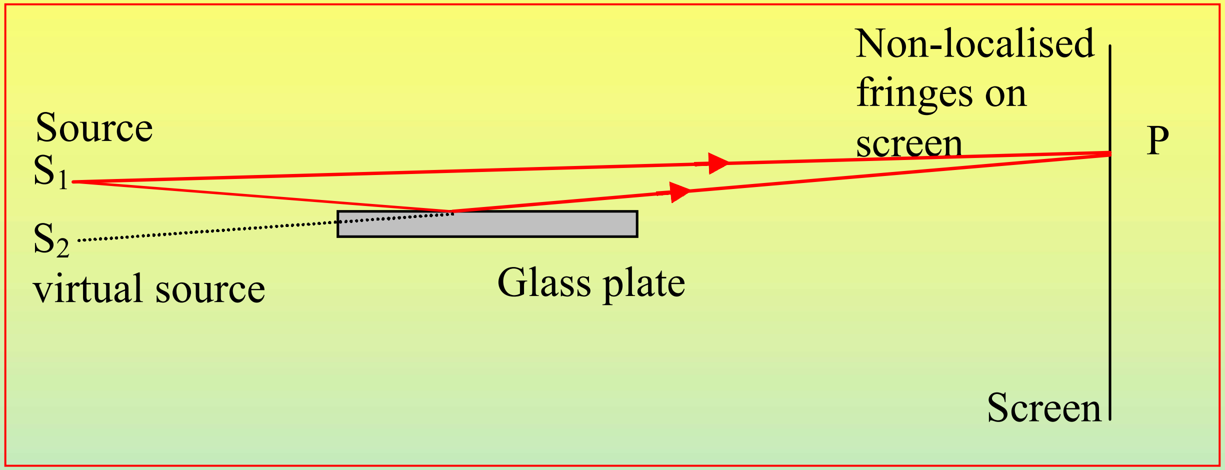


- You can see that the bigger the number of lines ‘n’ in the grating, the sharper the interference
  - the width before the irradiance falls to zero is just 360°/n
    - e.g. n = 40,000 , the width is 9×10^{-3} degrees
- the peaks are so narrow that each spectral line forms its own isolated fringe
  - the separate fringes are known as the *first order spectrum*, the *second order spectrum*, etc.
- the irradiance from the grating increases as n^2



Cd Spectrum

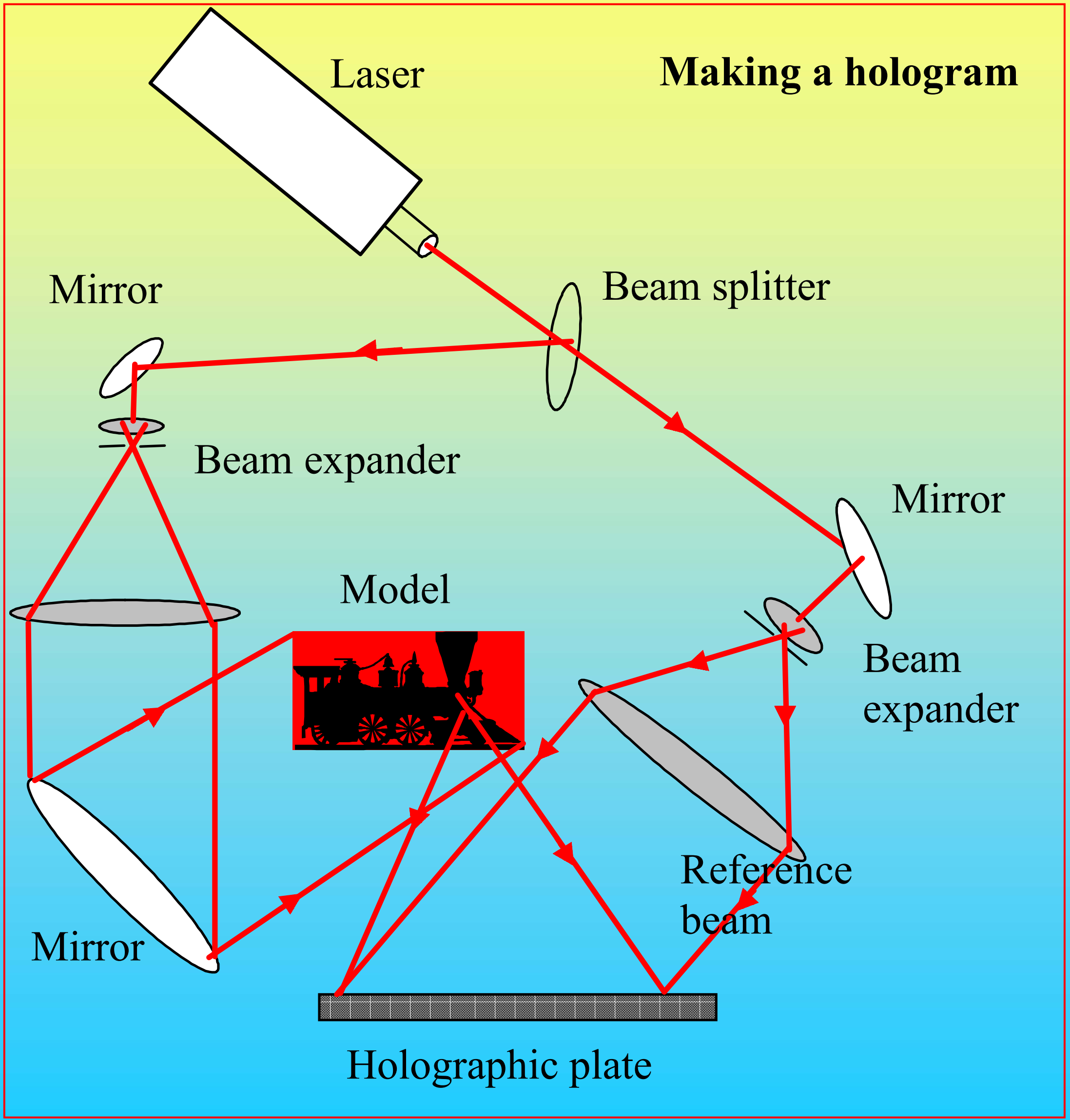
Lloyd's mirror



- Lloyd's mirror is a variant on Young's slits that is of interest because
  - it is brilliantly simple
  - it shows that light reflected from a more dense medium undergoes a phase change of π (180°)
  - the arrangement is very close to that needed to make a hologram, though it is 100 years older

Making a hologram

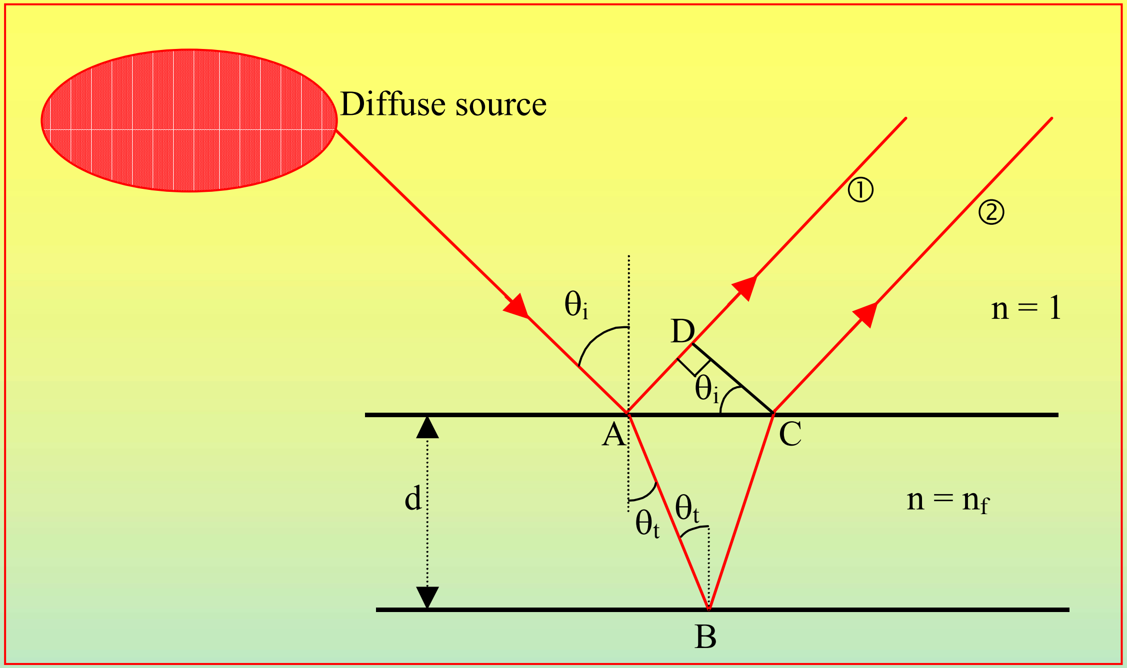
- A hologram is a record of the interference pattern between a direct laser beam (the reference) and light from an object
- Viewing a hologram uses the principles of diffraction



Thin film fringes

$$2d\,n_f\cos\theta_t = m\lambda$$

m is the interference 'order'



- Thin film fringes are formed by the interference between light reflected from the top and bottom of a film: – division of amplitude
  - the film is often thin, but doesn't have to be
- Working out the extra path length taken by the light reflected from the bottom gives the condition for destructive interference shown above
  - extra path length is OPL(ABC) – OPL(AD) - λ/2

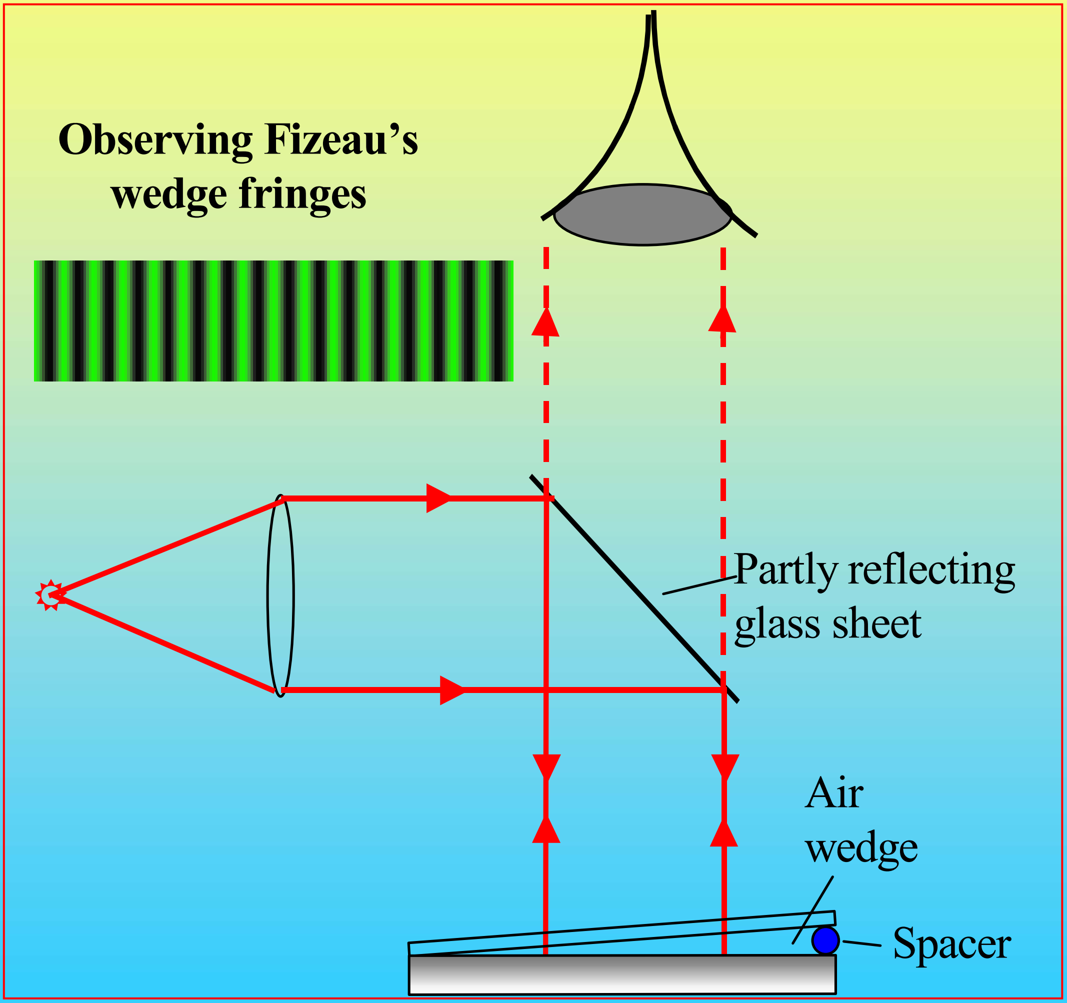
Fringes of constant inclination

- The colours on soap bubbles, oil on water, beetles backs and much more besides are examples of interference **fringes of constant inclination**

  - Haidinger's fringes, caused by the interference from either side of an *optical flat*, are observed as circular fringes when looking straight down on the flat
  - fringes of constant inclination appear to be located at ∞

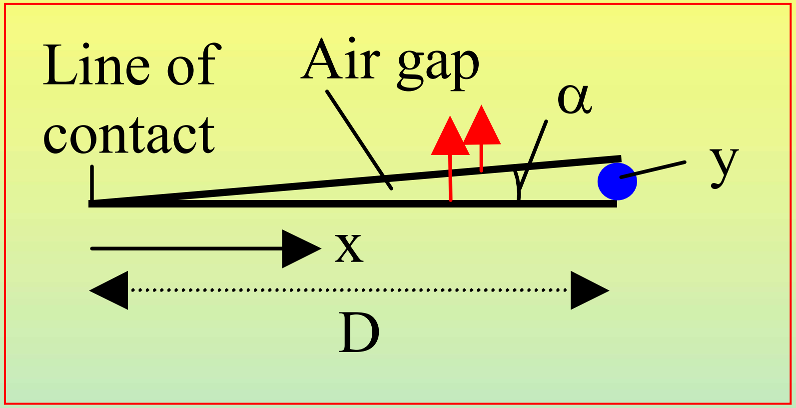
Fringes of constant optical thickness

- Fizeau's fringes obtained from an air wedge are a good example
- They are simple to set up and very useful for measuring the thickness of thin specimens
  - equi-spaced fringes are obtained, whose spacing can be measured with a low power microscope



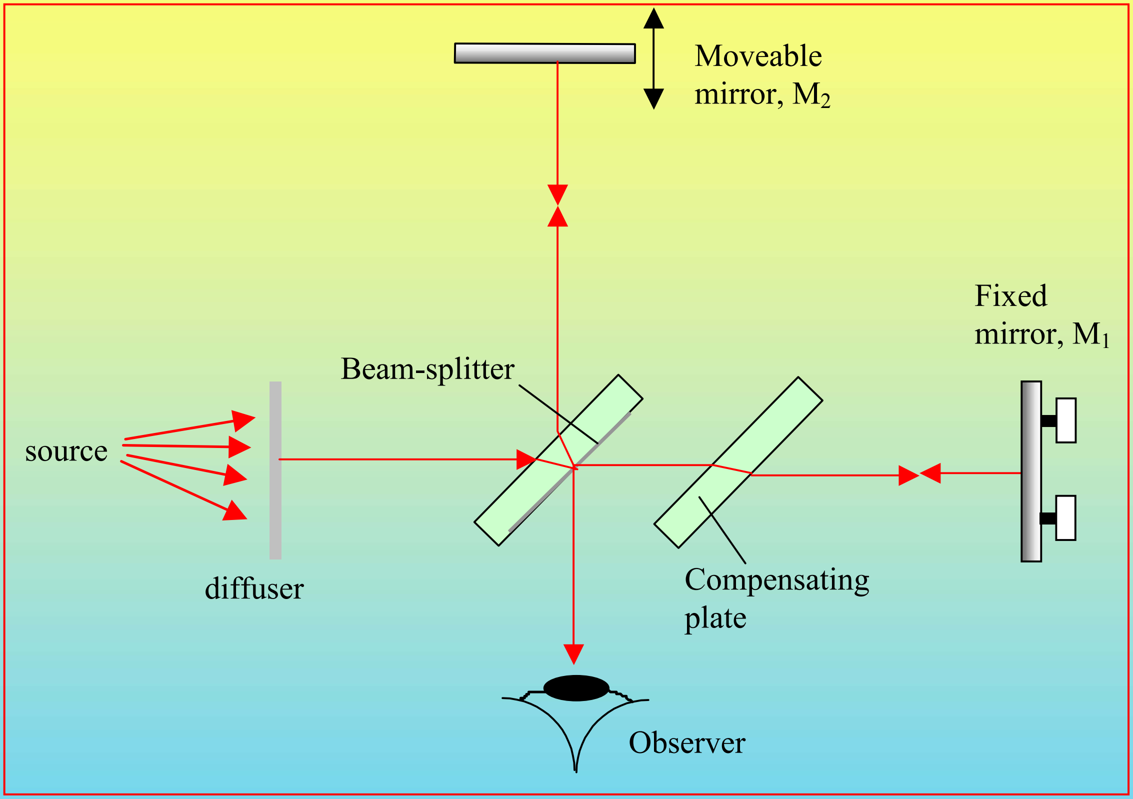
Working with Fizeau's fringes

- y is the spacer thickness
- D the distance between spacer and line of contact
- x the distance from line of contact to fringe
- α the angle of the wedge = y/D



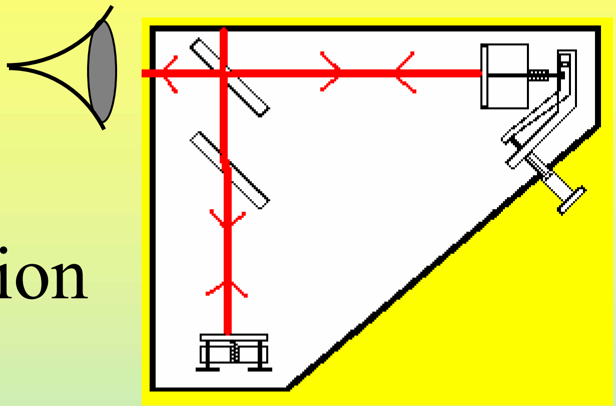
- From the previous result, when $\theta \approx 0$, or pretty obviously, the extra path difference is $2x\alpha$ ($+\lambda/2$ for the phase change on the lower reflection)
  - therefore $2x\alpha = m\lambda$ for a **dark** fringe
  - the separation of neighbouring fringes is $\Delta x = \lambda/2\alpha = \lambda/(2y/D)$
  - example: $\Delta x = 0.1$ mm; $\lambda = 500$ nm; D = 30 mm, then y = 75 µm

Michelson interferometer



- The Michelson interferometer is one of the great instruments of physical science
  - it is the archetype for other interferometers

What you see with the Michelson



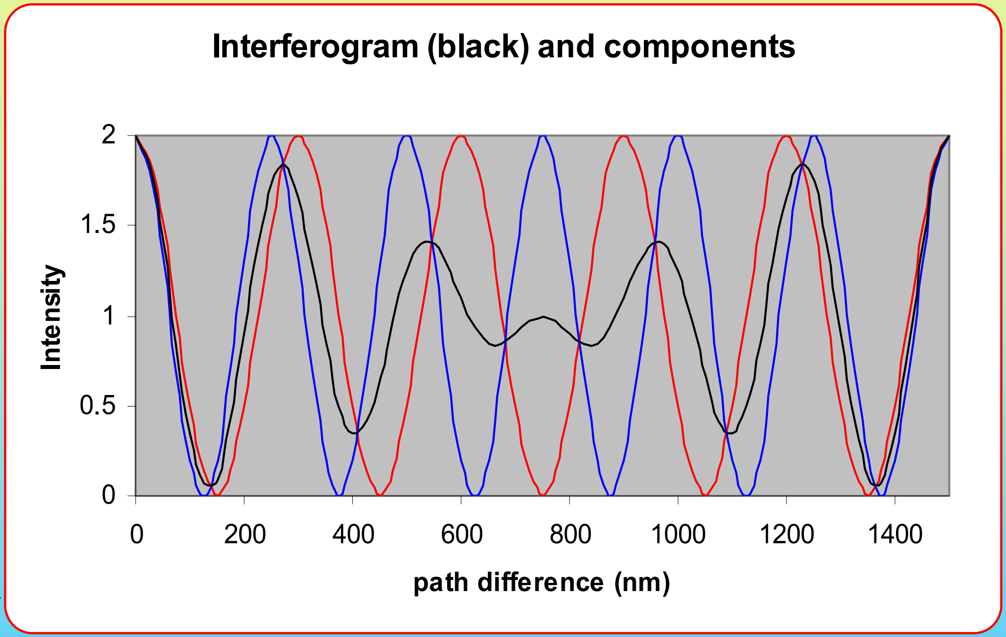
- With the mirrors parallel you see circular fringes of constant inclination
  - this is the most common way to use it
  - replacing your eye with a photocell, fringes can be counted
  - the motion of the moving mirror by $\lambda/2$ will shift the pattern by one complete fringe
    - detecting motion by 0.2 fringe is not hard, equivalent to a mirror movement of $\lambda/10 \approx 55$ nm for light in the middle of the spectrum
- With the mirrors inclined, straight Fizeau fringes are formed

What you can do with a Michelson

- Measure lengths (usually $\leq$ 1 m) to very high accuracy against an optical standard
- Measure movement of an object very accurately
- Measure position very precisely
- Compare the alleged flatness of an optical component against a standard flat mirror
- Use it as Fourier transform spectrometer to obtain high-resolution spectra

An interferogram

- An **interferogram** is a plot of the output of the interferometer as the path difference is changed
- The plot shows the output when the source contains two wavelengths, 500 nm and 600 nm



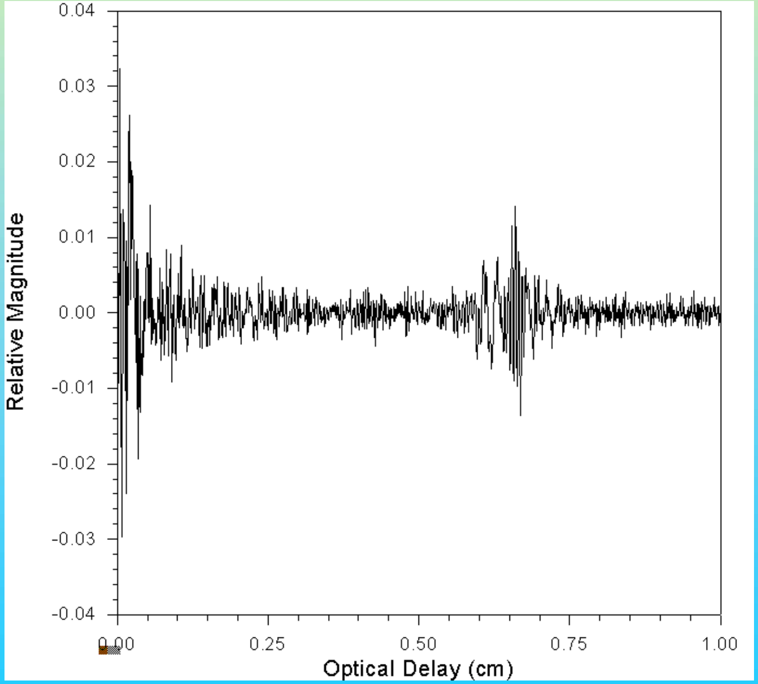
- Notice how the visibility fluctuates every 1500 nm change in the path difference of the arms

The Fourier transform spectrometer

- Each wavenumber in the incident light spectrum S(k) contributes its own variation in the interferogram of:

$$2S(k)(1+\cos(kx))$$

- The complete interferogram is therefore a sum of these cosine variations



- Mathematically, the spectrum can be recovered from the interferogram by the process of taking the *Fourier transform*