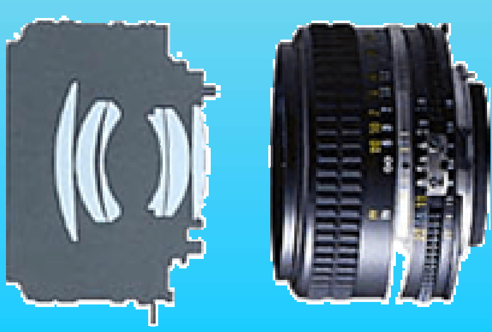
Stanley telescope

Imaging

Nikon waterproof binoculars

- This section covers
  - how images are formed
  - imperfect and perfect imaging
  - spherical image-forming surfaces
  - locating images
  - some imaging devices

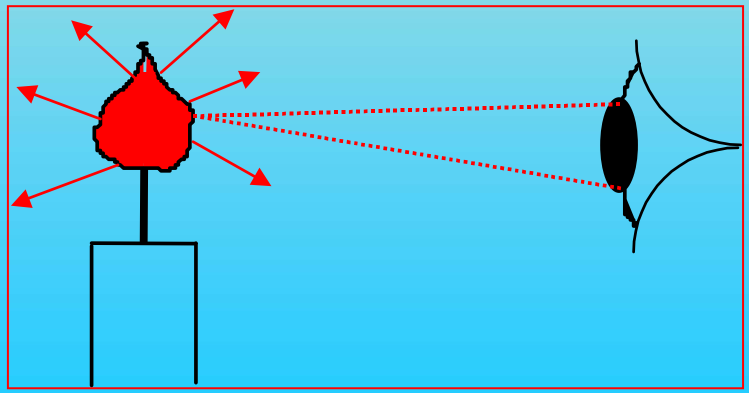


Nikkor 50 mm camera lens

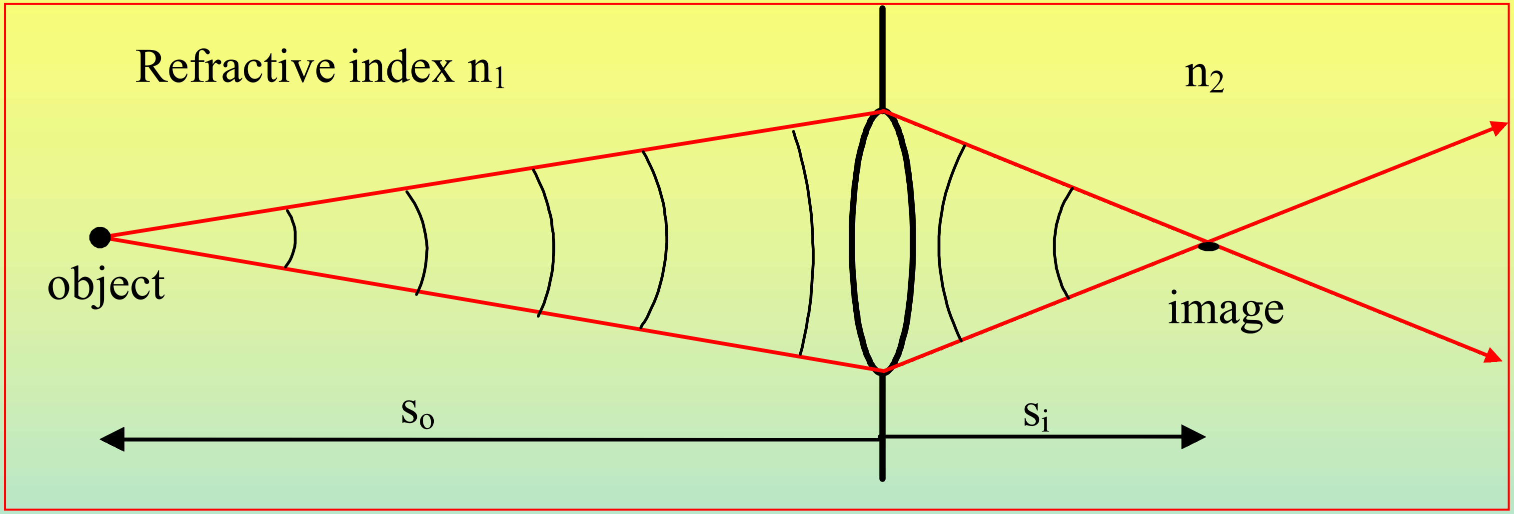
Olympus 2100 digital camera

Forming an image

- Each point of an object sends out light over a range of directions
- Seeing an object point requires intercepting a pencil of light with our eye
  - the point is located by us at the apex of the pencil
  - pencils are characterised by their **vergence**

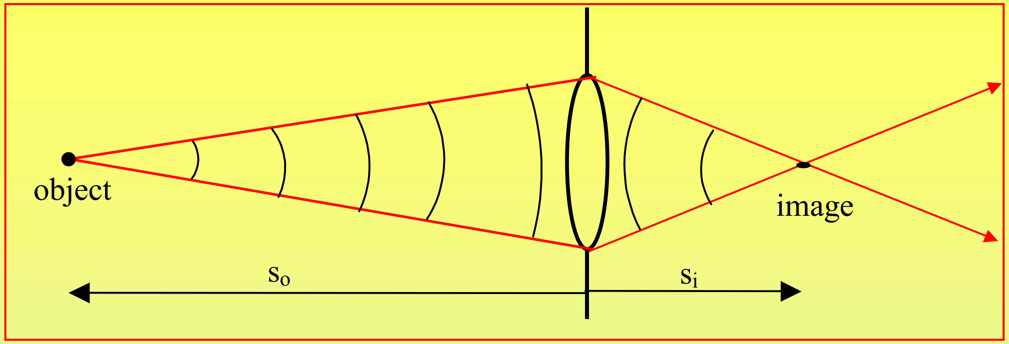


Vergence



- Vergence measures the convergence (+ve) or divergence (-ve) of a pencil of rays or, equivalently, a small section of travelling wavefront. At the aperture:
  - vergence of the wavefront forming the image = n_2/s_i
  - vergence of the wavefront from the object = $-n_1/s_o$
  - vergence is measured in **dioptres** ($\equiv m^{-1}$)

Effect of a lens



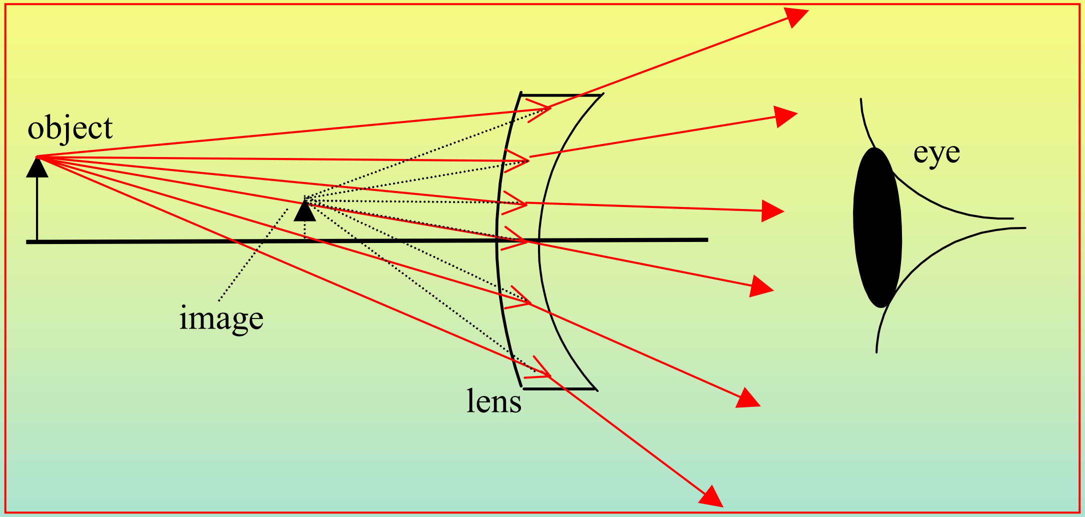
- A lens alters the vergence of a pencil by an amount equal to the **power** $\mathscr{D}$ of the lens

$$\frac{n_2}{s_i} = -\frac{n_1}{s_o} + \mathscr{D} \quad \text{or} \quad \frac{n_2}{s_i} + \frac{n_1}{s_o} = \mathscr{D}$$

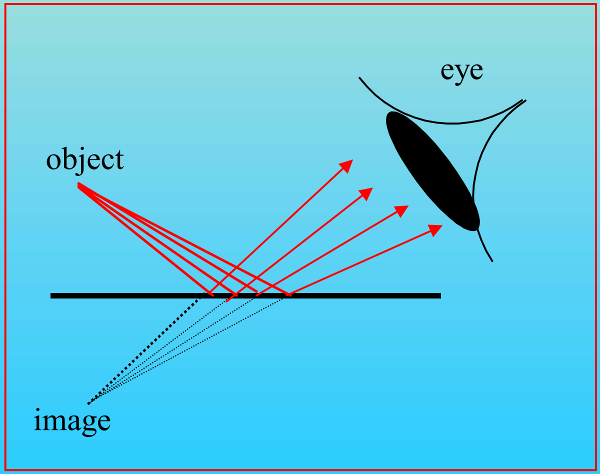
- **This is the fundamental imaging equation of a lens** (relating object and image distances)
  - remembering the relationship between rays and wavefronts, this is essentially a wave view of imaging

Graphic examples of imaging

- Imaging by a diverging lens
  - the vergence of the incident pencils is made more –ve by the lens



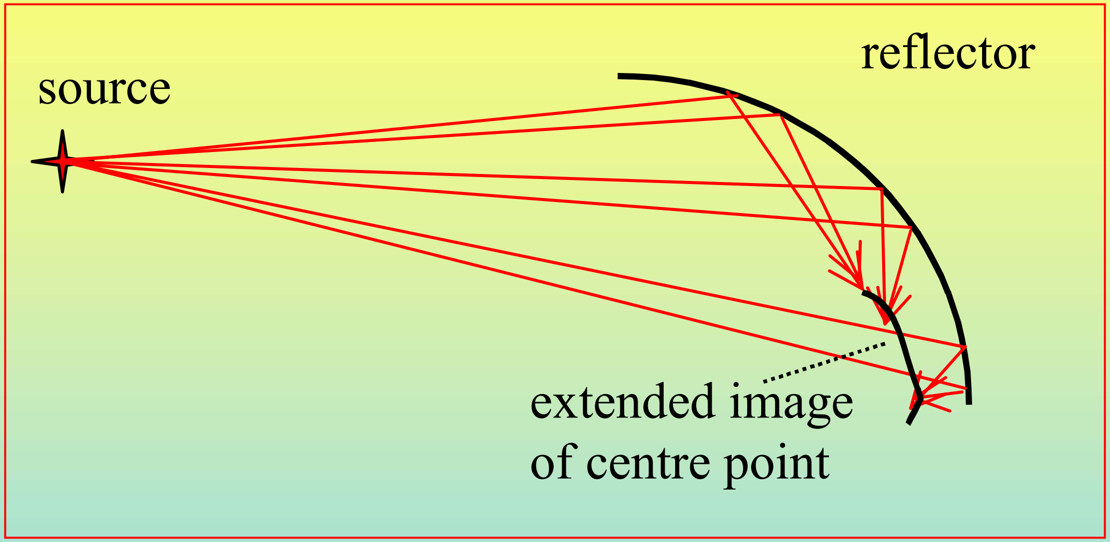
- Imaging by a plane mirror
  - there is no alteration of vergence, only a change in direction of the axis of the pencil



Summary

- Imaging requires the divergence or convergence of a bundle of neighbouring rays from each object point
- The apex of the pencil is the position of the image
- The image of an extended object is the composite image of all its points
- Each imaging surface takes for its object the image from the previous surface
- Simple image forming surfaces alter pencils of light in two ways:
  - they alter the vergence of the pencil
  - they may bend the axis of the pencil

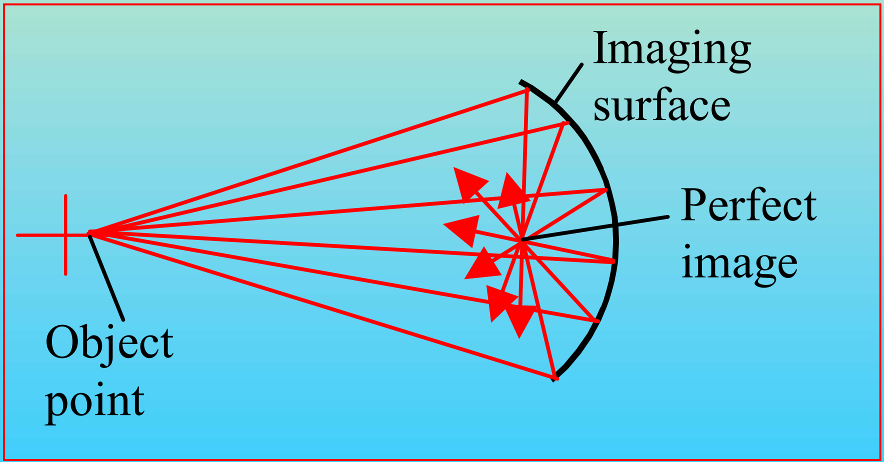
Imperfect imaging



- Different parts of an imaging forming surface may form images in different places
  - the result is a blurred image
  - the blur reduces when a smaller amount of image forming surface used

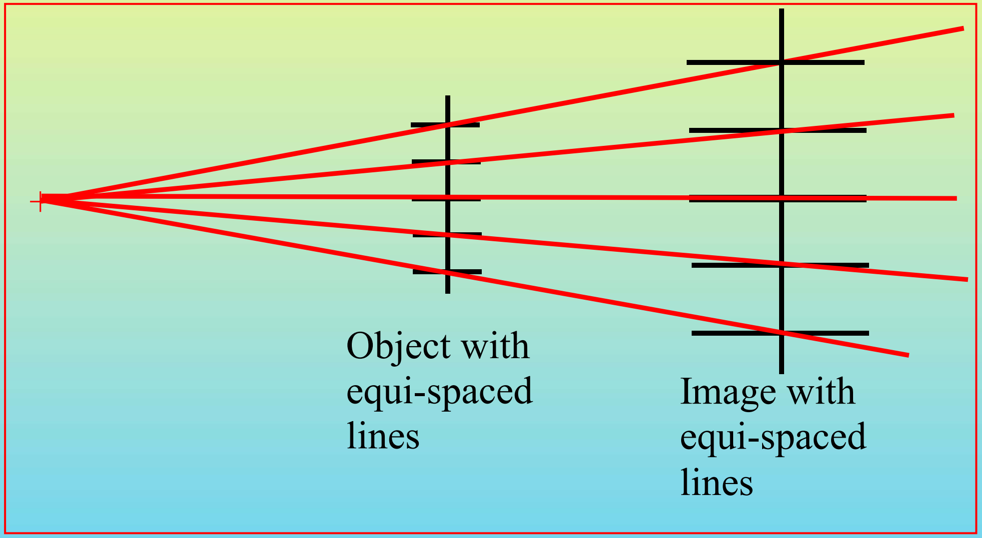
Perfect imaging - 1

- A perfect image forms when every part of the image-forming surface produces an image at the same place
- Mathematically, a perfect image creates a one-one mapping from **object space** to **image space**
- Image point and object point are **conjugate**

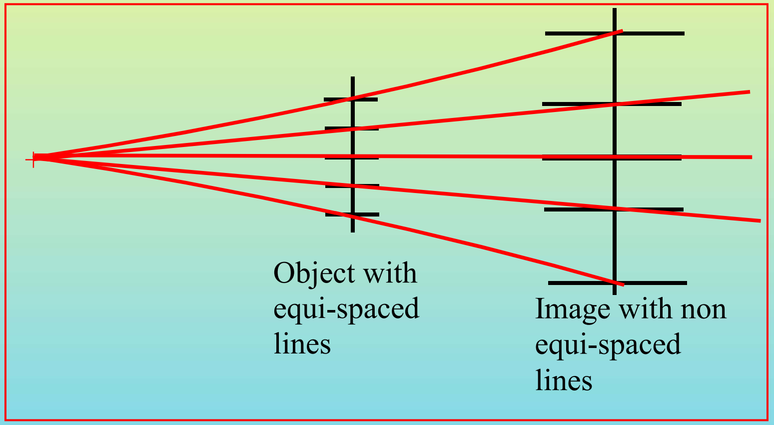


Perfect imaging - 2

- A perfect image mapping must be **linear**



Linear mapping

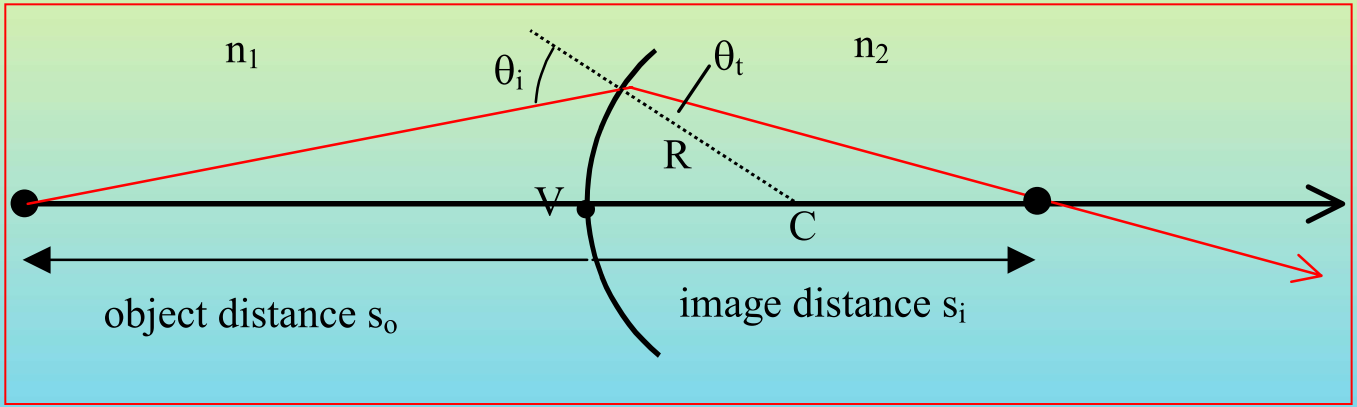


Non-linear mapping

- Perfect imaging only happens with a plane mirror

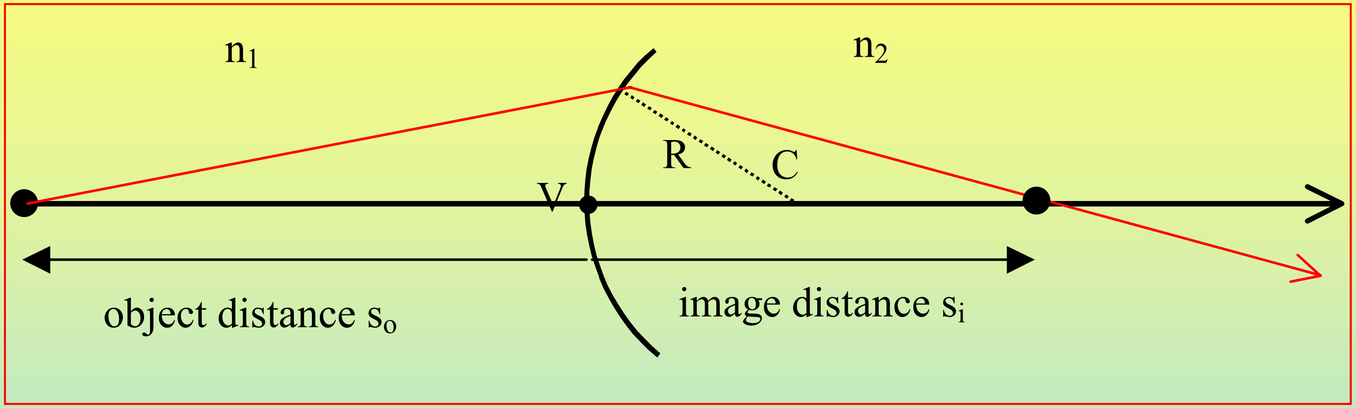
The paraxial approximations

- Angles of incidence, θ_i , and refraction, θ_t , are small



- Off-axis distances are small compared with the curvature of the refracting surface and with object and image distances (R, s_o, s_i)

Spherical surfaces

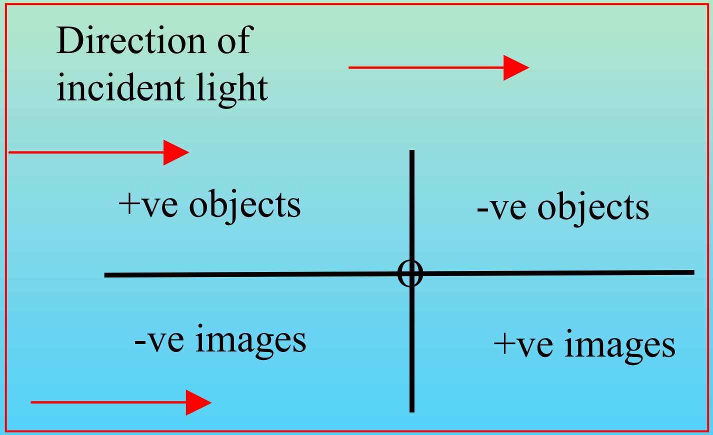


- Under the **paraxial approximations** spherical surfaces form ideal images
- All rays from a single object point pass through a single image point after refraction

$$\frac{n_2}{s_i} + \frac{n_1}{s_o} = \frac{n_2 - n_1}{R} = \mathscr{D}$$, the power of the surface

Sign convention

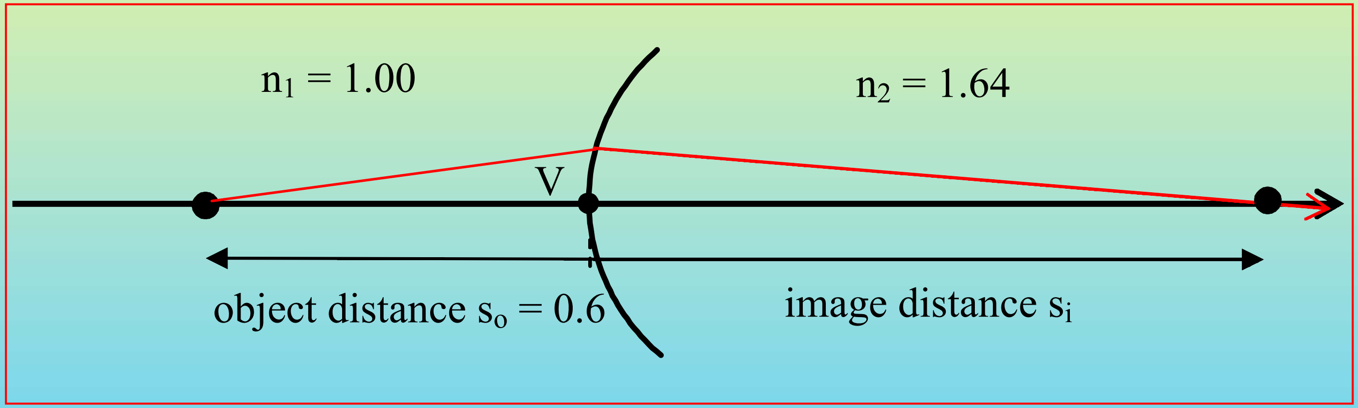
- For an algebraic equation to give the right answer when images (and objects) can be on either side of a lens you need a **sign convention** to determine which quantities are +ve and which are –ve
  - **real** objects are those in front of the image forming surface, in front being taken with respect to the direction of incident light
  - **virtual** objects are those after the image forming surface
  - the reverse is true for images



- The sign convention is known as **real is positive**

Worked example

- An object is placed on the axis of a refracting surface of radius of curvature 200 mm; the object is 600 mm from the surface in air (refractive index 1.00) and the surface has refractive index 1.64. *Where is the image?*

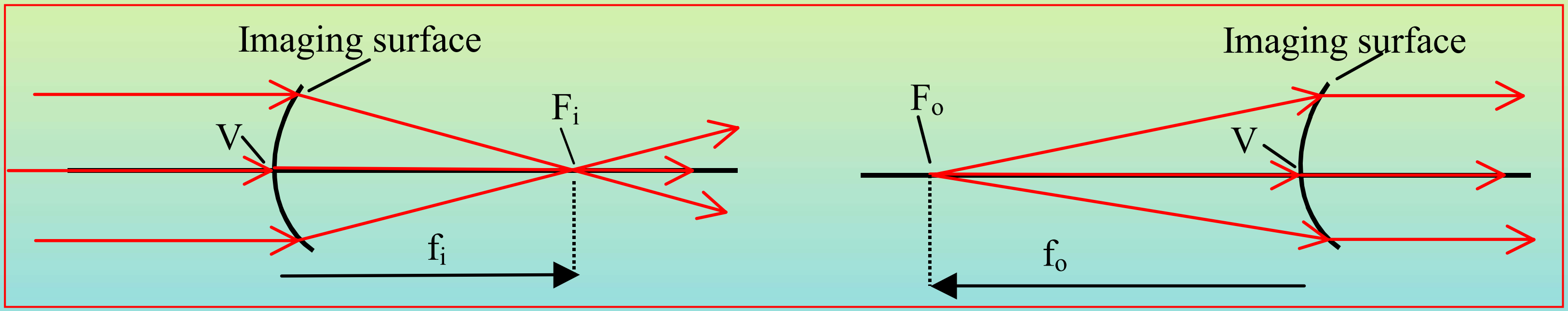


- Given: $n_1 = 1.000$; $n_2 = 1.64$; $R = 200$ mm; $s_o = 600$ mm $\equiv 0.6$ m
- Surface power $\mathscr{D} = (n_2 - n_1)/R = (1.64 - 1.00)/0.2 = 3.2$ dioptres
- Find s_i from $\frac{n_2}{s_i} + \frac{n_1}{s_o} = \mathscr{D}$. Therefore $\frac{1.64}{s_i} + \frac{1.00}{0.6} = 3.2$ $s_i = 1.07$ m

Focal points and lengths

Nikon underwater lens

- An imaging surface has 2 focal points
  - F_i in image space and F_o in object space



- F_i is the image point of an axial object at ∞
- F_o is the axial object point whose image is at ∞
- $f_i = VF_i$, the image focal length
- $f_o = VF_o$, the object focal length

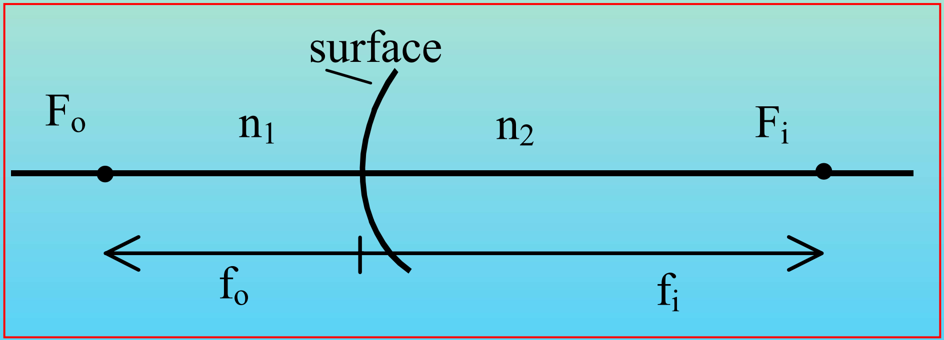
Where are the focal points?

- The imaging equation tells us where the focal points are
  - for F_i, $s_o = \infty$; $\frac{n_2}{s_i} + \frac{n_1}{s_o} = \frac{n_2 - n_1}{R} = \mathscr{D}$, the power of the surface

$$\frac{n_2}{f_i} + \frac{n_1}{\infty} = \frac{n_2 - n_1}{R} = \mathscr{D} \quad \text{hence} \quad \frac{n_2}{f_i} = \frac{n_2 - n_1}{R} = \mathscr{D}$$

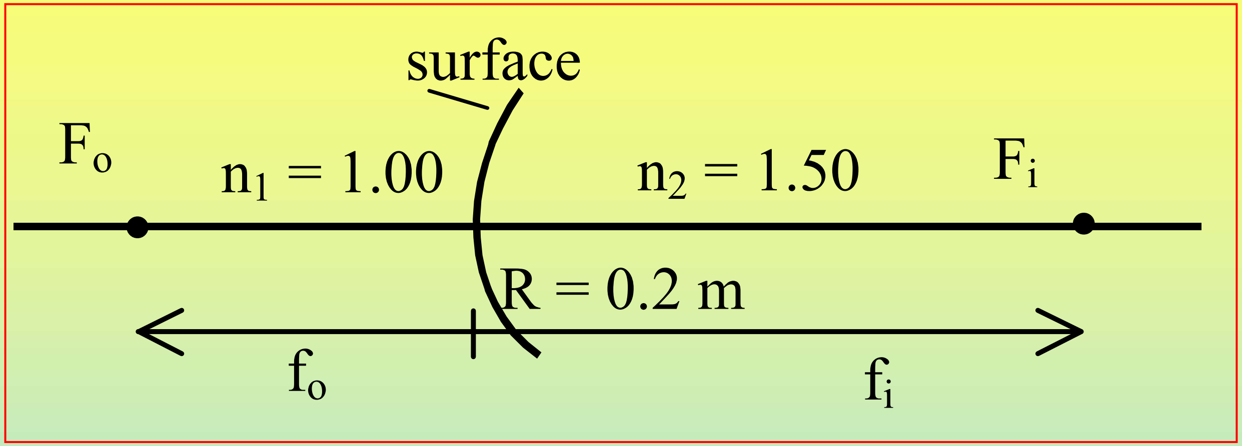
  - likewise $\frac{n_1}{f_o} = \frac{n_2 - n_1}{R} = \mathscr{D}$

- F_o and F_i are on either side of the surface



- For a given power $\mathscr{D}$, f depends on the refractive index of the medium

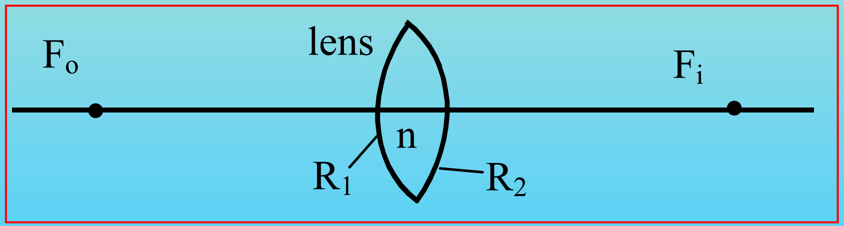
Focal point example



- An imaging surface between media of refractive indices 1.00 and 1.50 has radius of curvature 200 mm. *What is the power of the surface? What are its focal lengths?*
  - $\mathscr{D} = (1.50 - 1.00)/0.2 = 2.5$ dioptres
  - $f_i = 1.50/\mathscr{D} = 0.6$ m; $f_o = 1.00/\mathscr{D} = 0.4$ m

The usefulness of focal points

- If you know the focal lengths for a surface, and one other fact about it, e.g. its radius of curvature, you can calculate all its imaging
- Focal points are 2 out of 6 cardinal points that characterise an imaging component
- A lens is simply two imaging surfaces, one after the other



  - a lens is specified by its rear (image) focal length
  - focal points are usually equi-spaced from the centre

Imaging equation for a thin lens

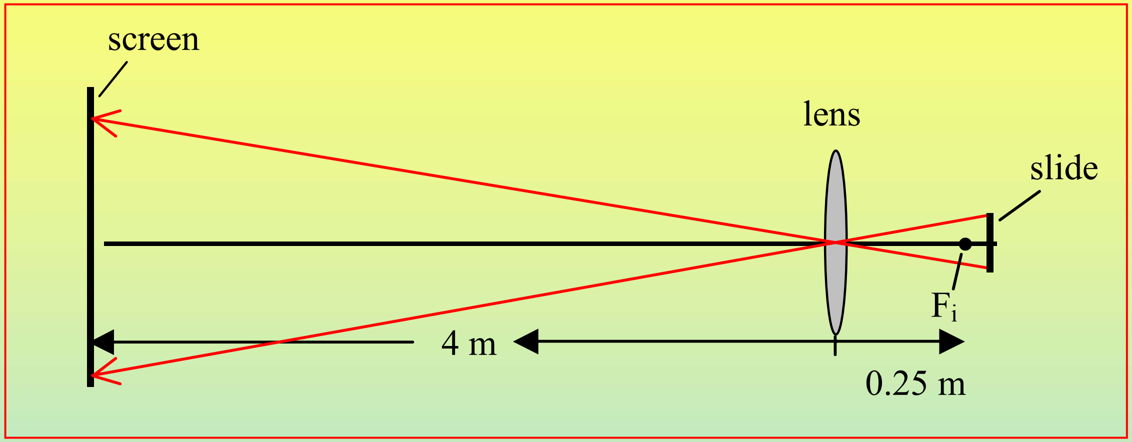
- Application in succession of the single surface equation to each surface of a thin lens gives

$$\frac{1}{s_o}+\frac{1}{s_i}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}=\mathscr{D}$$

- This is known as the '**thin lensmaker's equation**'

Olympus tele-converters

Lensmaker's equation example



- An image of a slide is to be formed 4 m in front of a lens of focal length 250 mm. *How far from the lens must the object be placed?*

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}, \quad \textit{therefore} \quad \frac{1}{s_o}+\frac{1}{4}=\frac{1}{0.25}, \textit{ giving } s_o=\frac{4}{15}\,m=0.267\,m$$

Example of an enlarger

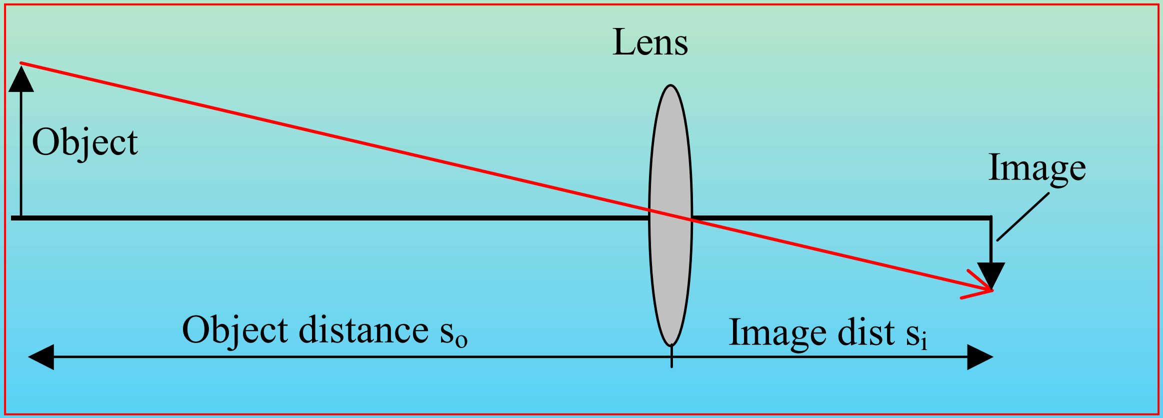
- An enlarger lens has a focal length of 50 mm and is positioned 60 mm beneath the negative being enlarged. *How far below the lens must the printing paper be placed so that the image is in focus?*
- $f = 50$ mm; $s_o = 60$ mm; s_i ?

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}, \quad \textit{therefore} \quad \frac{1}{60} + \frac{1}{s_i} = \frac{1}{50}, \textit{ giving } s_i = 300\ mm$$

- Notice that so long as the same units are used for all the known distances, the result is in these units

Transverse magnification

- The fact that a light ray passes through the centre of a thin lens undeviated, makes the magnification simply the ratio of image distance to object distance
  - remembering the sign convention, that gives image and object heights different signs in the diagram



$$\text{Magnification} = \frac{\text{Image height}}{\text{Object height}} = -\frac{s_i}{s_o}$$

Examples

Referring back to the previous questions

- *What magnification did the slide projector give the projected picture?*

  - $s_i = 4$ m; $s_o = 4/15\ m$, hence,

Magnification = $-s_i/s_o = \times -15$

- *What magnification did the enlarger produce of the negative?*

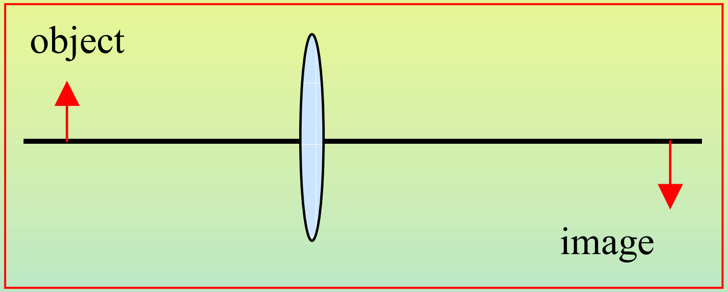
  - $s_i = 300\ mm$; $s_o = 60\ mm$, hence

Magnification = $-s_i/s_o = -300/60 = \times -5$

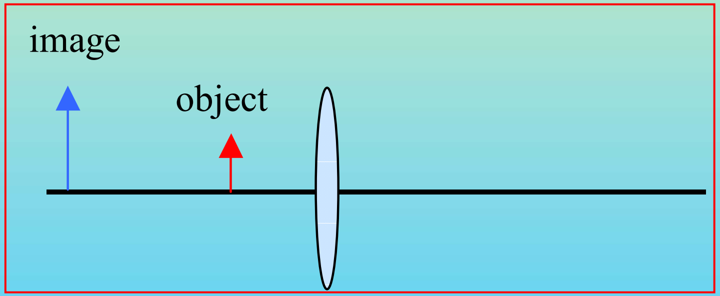
  - the –ve signs show that the images are upside down

Recap of sign convention

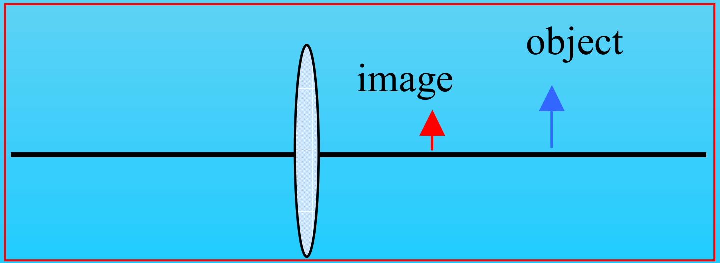
- Measurements are in object space and image space
- **Real** object **+ve** s_o
- **Real** image **+ve** s_i



- **Real** object **+ve** s_o
- **Virtual** image **-ve** s_i

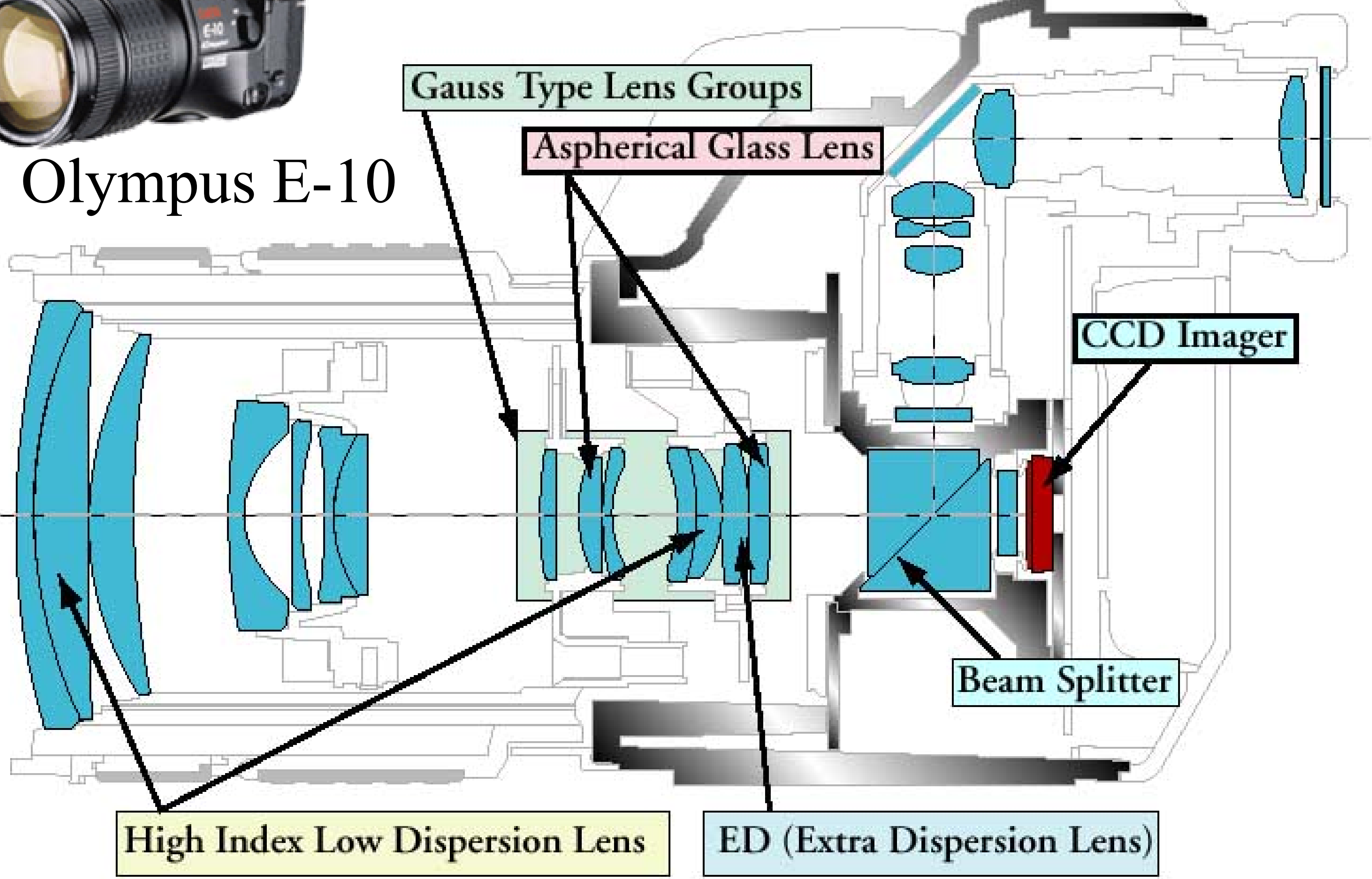


- **Virtual** object **-ve** s_o
- **Real** image **+ve** s_i



eveloped 14 element, 11 group 4 X zoom for the E-10,
very high precision and highly accurate optical system.

Olympus E-10



Courtesy: imaging-resource.com

Bushnell

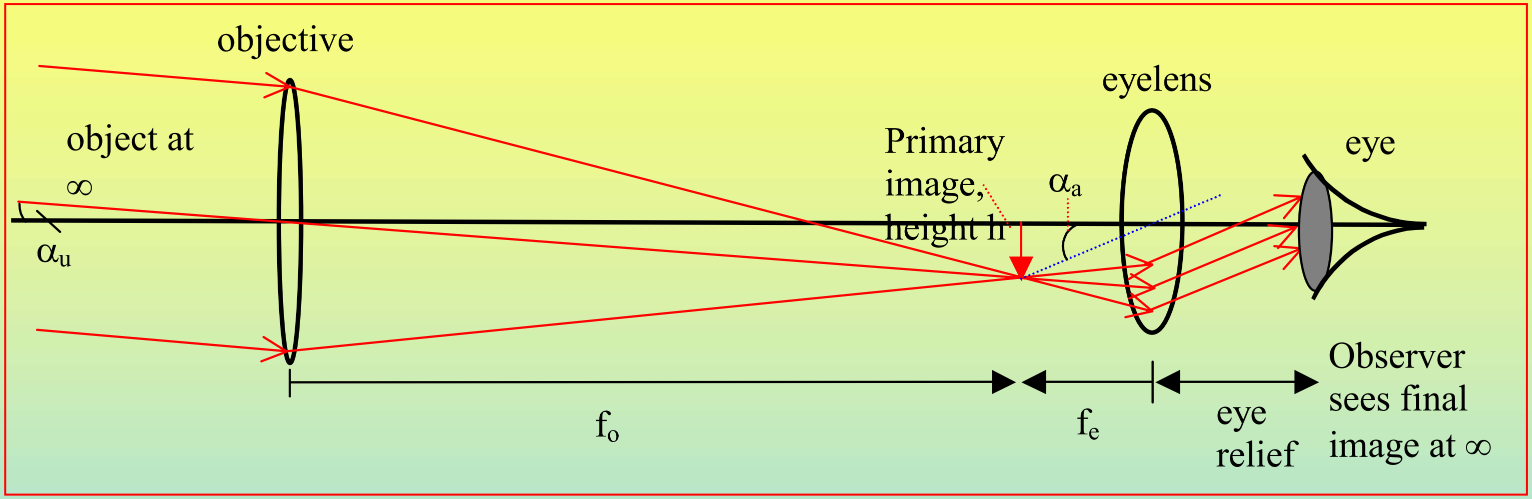
Telescopes

Leica

- Telescopes are the prime instruments of astronomy
- In the form of binoculars, they are used widely by naturalists, sailors and many other groups
- Telescopes are found widely, embedded in other instruments such as theodolites and spectrometers

TV102

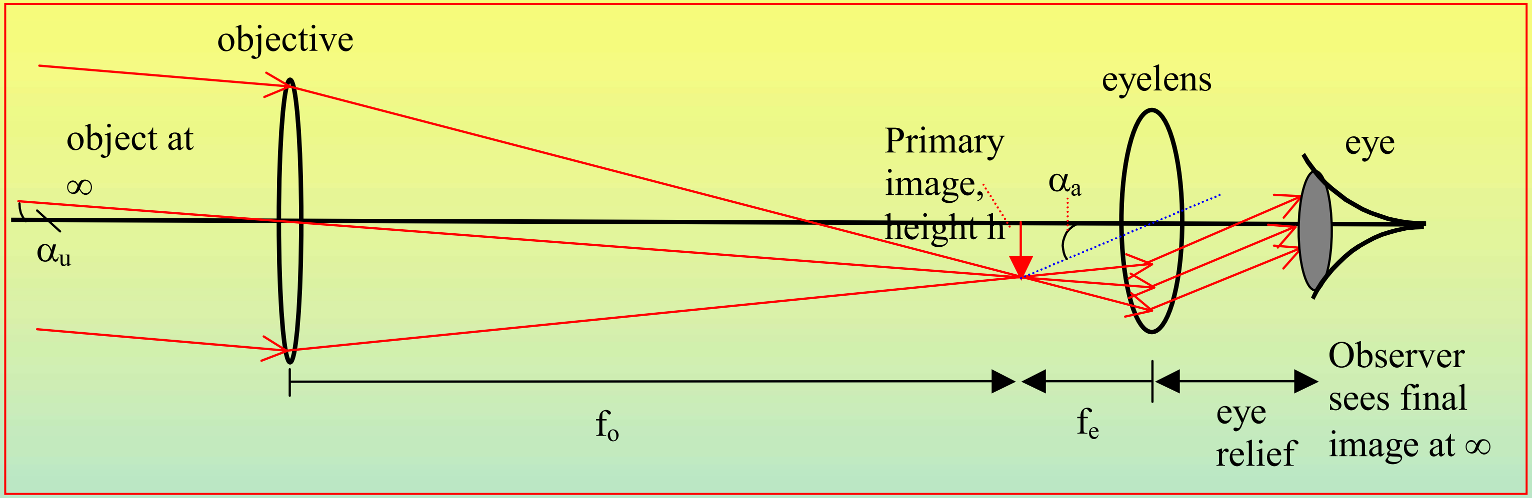
The basic telescope - Kepler



- The **objective** forms an image of an object at infinity
- The **eyelens** forms a virtual image at infinity of the primary image

Griffith Observatory refractor

Telescope magnification



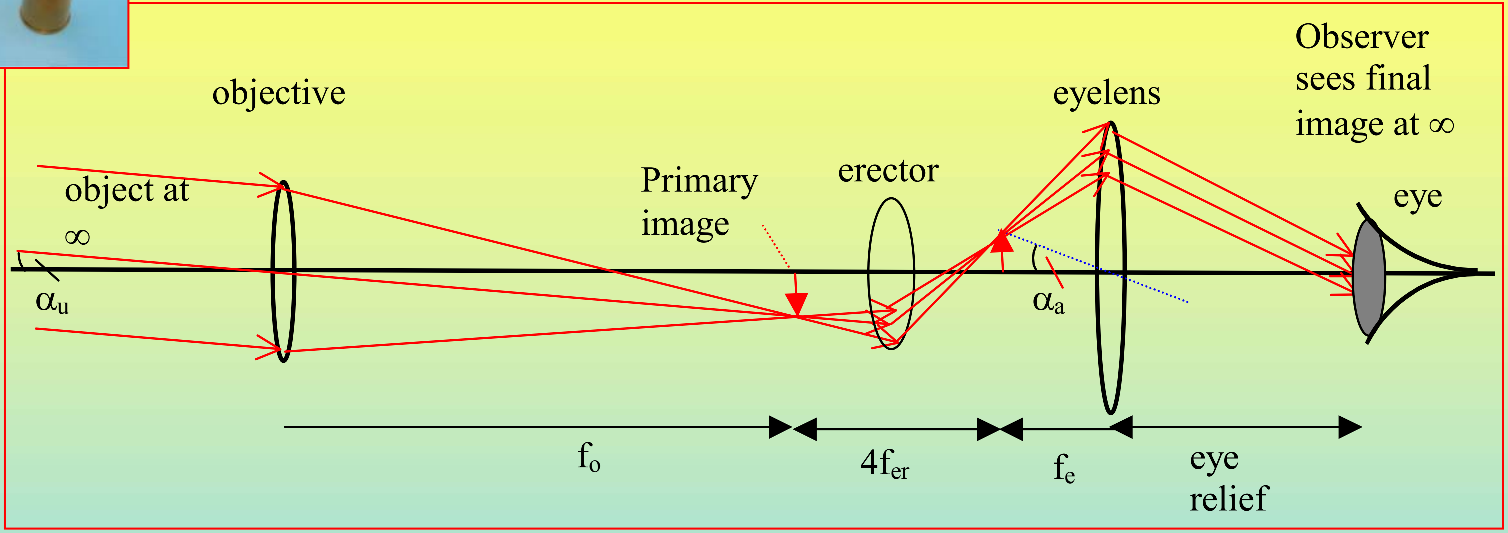
$$\text{angular magnifying power}\,(MP) = \frac{\text{angle subtended by image at eye}\,(\alpha_a)}{\text{angle subtended by object at eye}\,(\alpha_u)}$$

$$\alpha_a \approx \tan\alpha_a = \frac{h}{f_e}\text{, in the paraxial approximation}$$

$$\alpha_u \approx \tan\alpha_u = -\frac{h}{f_o}$$

$$\therefore MP = -\frac{f_o}{f_e}\text{, the } -\text{ve sign indicating an inverted image}$$

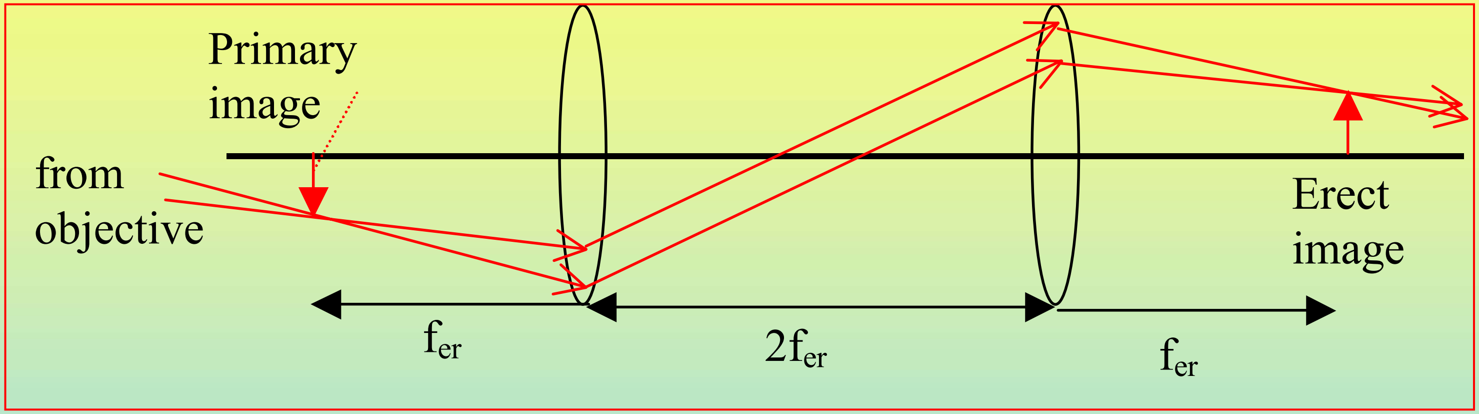
Erecting telescope



- In principle, image erection can be provided by a single lens
  - the eye relief has got larger
  - the erector limits the field of view
  - a larger eyepiece diameter is needed

Stanley "spyglasses"

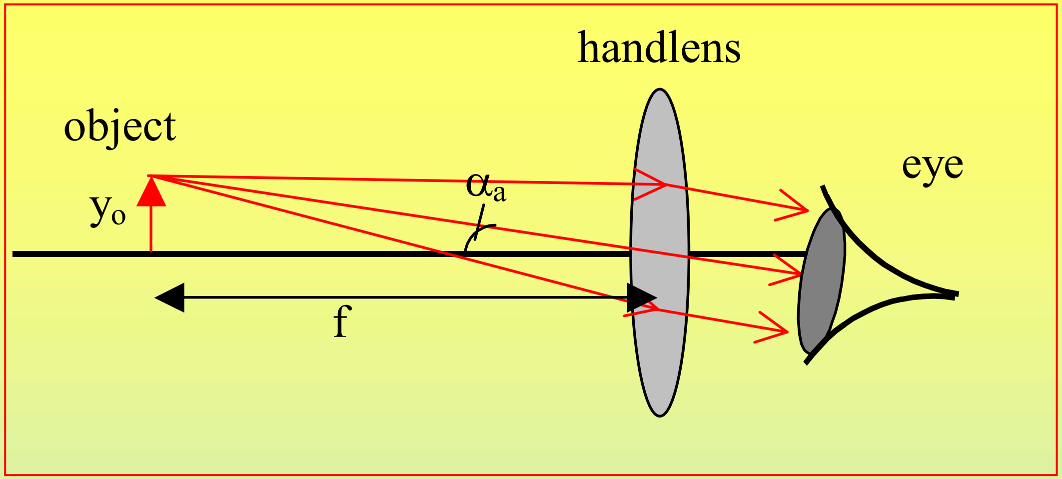
Rheita’s erector; prism erector



- In practice, erection with lenses is achieved using 2
- The arrangement is really a telescope of magnification ×-1 within a telescope
- Even more common is the use of a dual prism erector

Simmons spotter scope

Simple eyepiece



- The closest distance that we can see objects clearly is called **the nearest distance of distinct vision**, d_o (~0.25 m)
- The simple handlens forms a virtual image at infinity, which a relaxed eye can focus on
- Magnifying power:

$$MP = \frac{\alpha_{a(\text{aided})}}{\alpha_{u(\text{unaided})}}$$

- Since $\alpha_a = \frac{y_o}{f}$; and $\alpha_u = \frac{y_o}{d_o}$,

$$MP = \frac{d_o}{f} = 0.25\mathscr{D}$$

Example of handlens magnification

- *What is the power of a "×10" eyepiece?*
- Use:

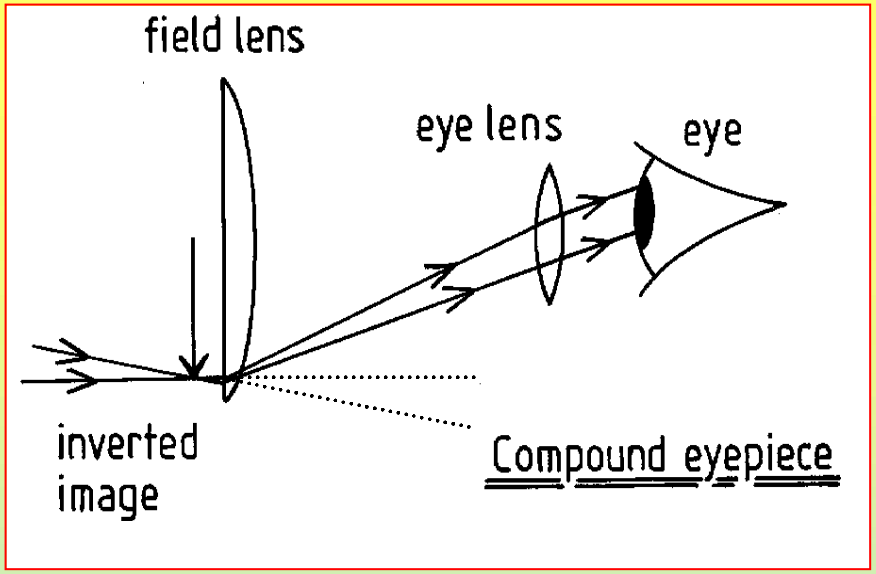
$$MP = \frac{d_o}{f} = 0.25\mathscr{D}$$

therefore: $10 = 0.25\,\mathscr{D}$

hence $\mathscr{D} = 40$ dioptres

Courtesy: wsu.edu

Compound eyepiece



- The diagram above, from the first year lab manual, introduces the **field lens**
  - bends round pencils of image-forming light so that they pass through a smaller following lens
  - improves image quality
  - allows control of the eye relief
  - clearly defines the field of view ("acts as a **field stop**")
- Real eyepieces contain 2 or more lenses

Telescope summary

- Telescopic systems are designed to have the object at ∞ and, for an observer, the final image at ∞
- The minimum telescope will have an achromatic objective and a multi-element eyepiece
- A terrestrial telescope will have a 2-lens or a prismatic erector
- The magnification is given by the ratio of focal lengths of objective to eyepiece
- Reflecting telescopes replace the objective lens by a mirror, which leads to a complete redesign of the system