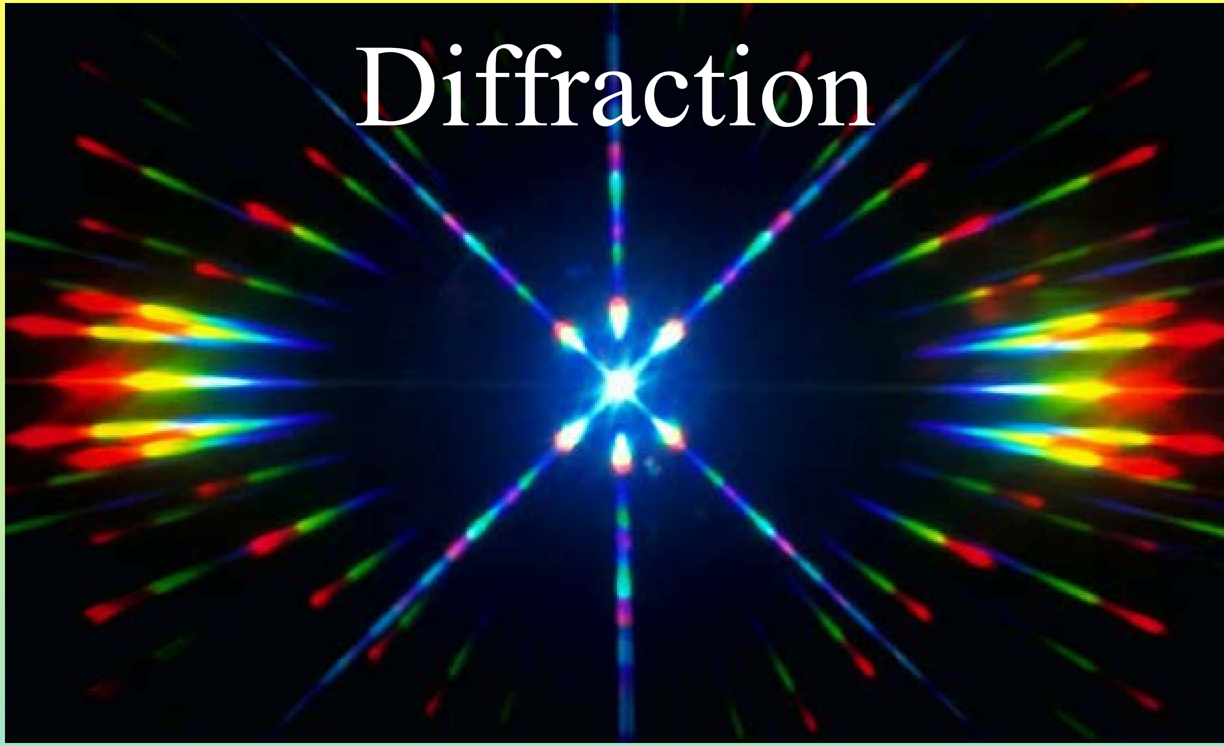
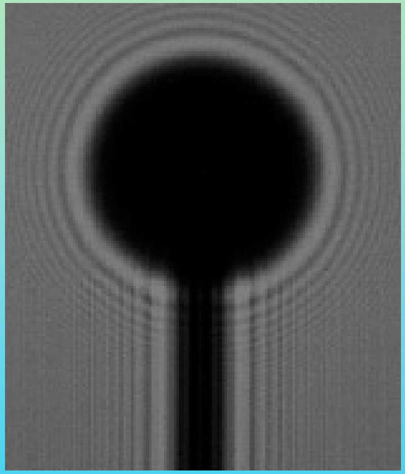
Diffraction



- Diffraction controls the detail you can see in optical instruments, makes holograms, diffraction gratings and much else possible, explains some natural phenomena
- Diffraction was discovered by Franceso Grimaldi in the first half of 17^{th} century
  - modern investigations date from Augustin Fresnel

What is diffraction?

- Diffraction is the spreading out of light from its geometrically defined path
  - diffraction is characteristic wave behaviour
- Diffraction typically appears as dark and bright fringes
- The underlying cause is the addition of waves from a continuous line or surface of sources



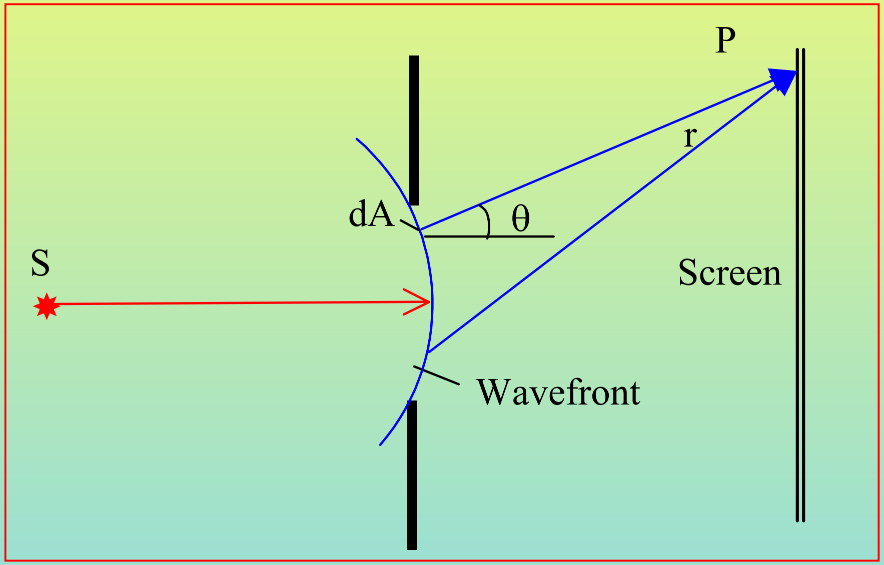
Diffraction around a round pin head

Huygens – Fresnel – Kirchhoff

- **Huygens** principle underlies the idea that each point on a wavefront acts as a source of secondary wavelets
- **Fresnel** put this into mathematical form, integrating the appropriate Ecos(kr - ωt) contributions
- **Kirchhoff** put in place all the correct multiplying terms

How it works

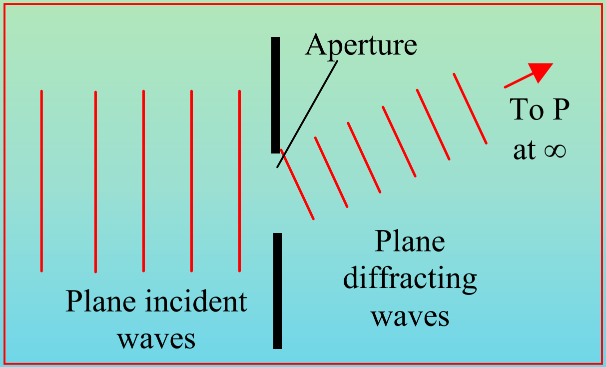
- Diffraction occurs when an advancing wavefront is partially blocked by an aperture
  - apertures we'll consider are slits, circles or rectangles



- Each area dA on the advancing wavefront acts as a source that radiates to P
- The total illumination at P is the **integral** over all the wavelets from within the aperture reaching P
- Evaluating this integral in general determines the **Fresnel diffraction pattern**
  - it is not particularly simple to find the result

Fraunhofer diffraction

- The most useful diffraction to look at is the special case of Fraunhofer diffraction
- There are 2 simplifying approximations
  - the source is at ∞
  - the pattern is at ∞
    - therefore the phase change across the source of the contributing waves varies linearly with position
  - mathematically:
    - remember:

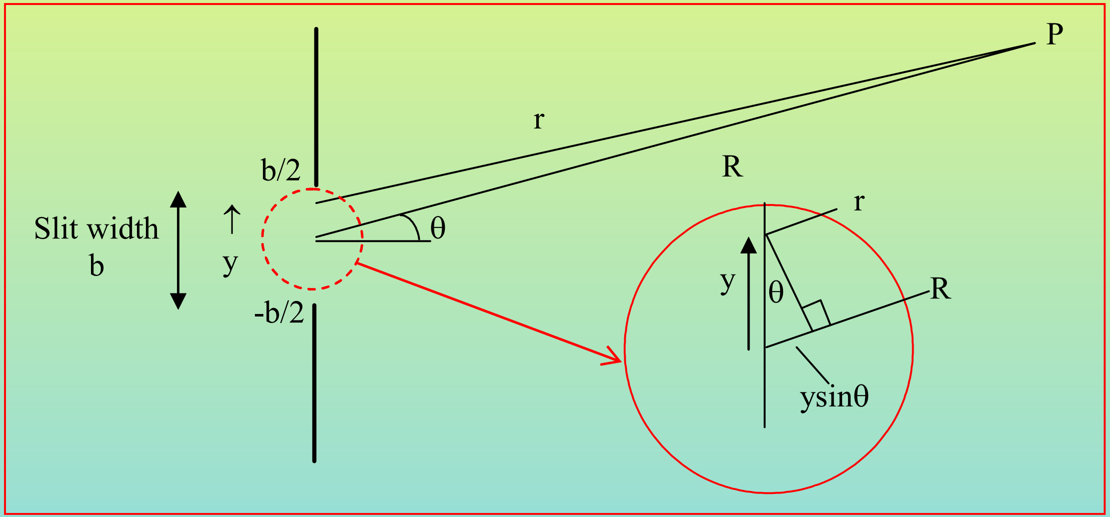


$$E_P \propto \int_{aperture} \cos(kr - \omega t)\, dA$$

$$I \propto \langle E_P^2 \rangle$$

Origin of Fraunhofer diffraction from a slit

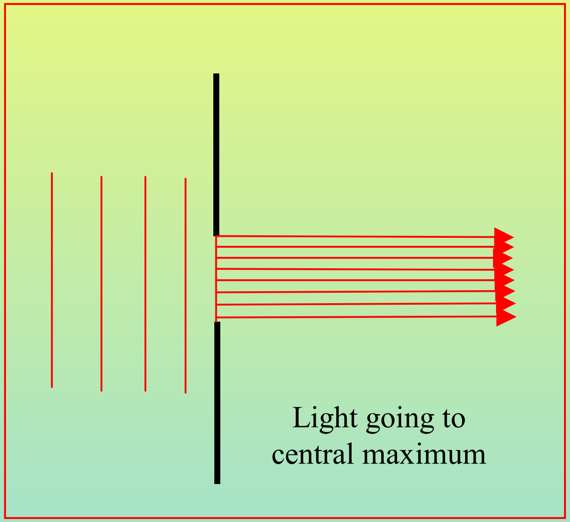
- The extra path length from the middle of the slit is $y\sin\theta$
- You have to integrate (sum) all the contributions to the wave reaching P from across the slit
  - the integration runs from –b/2 to +b/2

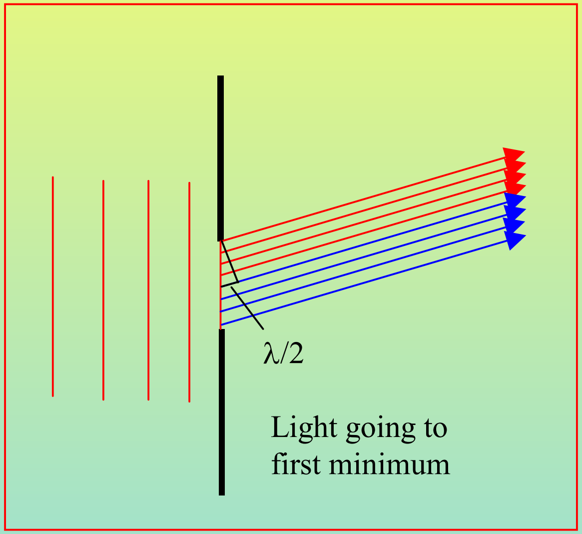


The slit

The diffraction pattern

Seeing what happens





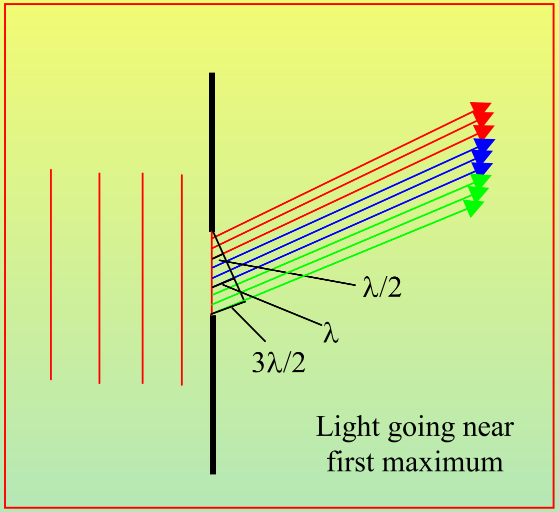
- At the first minimum, light from the middle of the slit is λ/2 out of phase with light from the top
  - the lower half light cancels out with the top half
- The angle at which the first minimum occurs

$$(b/2)\sin\theta = \lambda/2$$

or

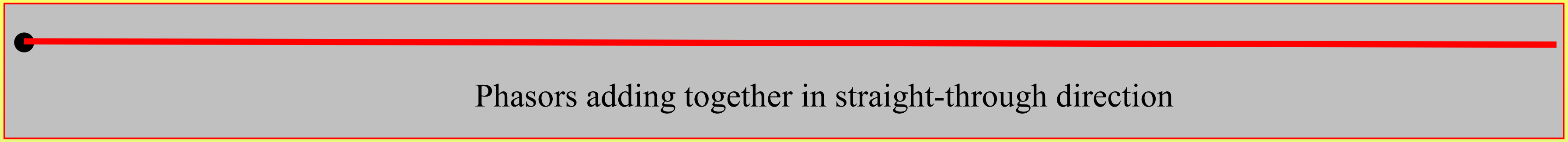
$$b\sin\theta = \lambda$$

Farther out



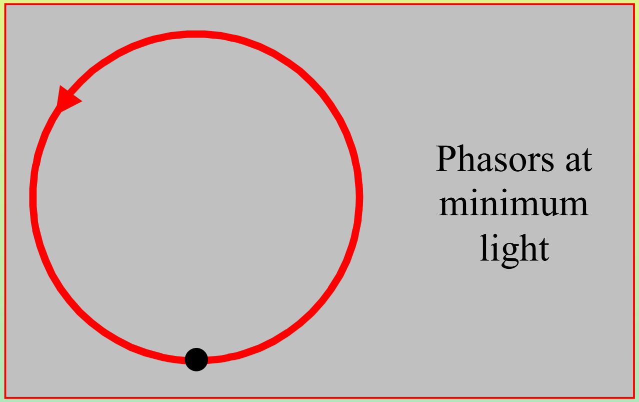
- Near the first maximum, light from the first third is cancelled by light from the second third, leaving the final third
- At a bigger angle θ, light from the first quarter is cancelled out by light from the second quarter and the same for the next two quarters
  - there are therefore minima (zeros) when

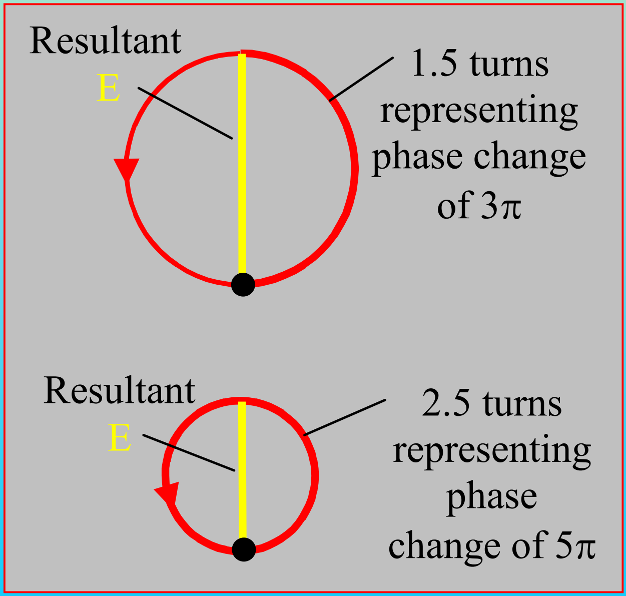
$$b \sin\theta = n\lambda \text{ , } n \text{ an integer}$$



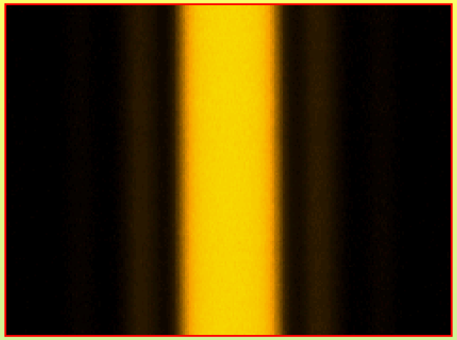
Phasor picture

- Imagine the slit divided into very many small segments
- The diagrams show
  - central maximum
  - first minimum
  - first off-centre maximum
  - second off-centre maximum





Quantitative expressions



- The integration for E_P gives:

$$E_P \propto b\frac{\sin\beta}{\beta}, \text{ where } \beta = k\frac{b}{2}\sin\theta$$

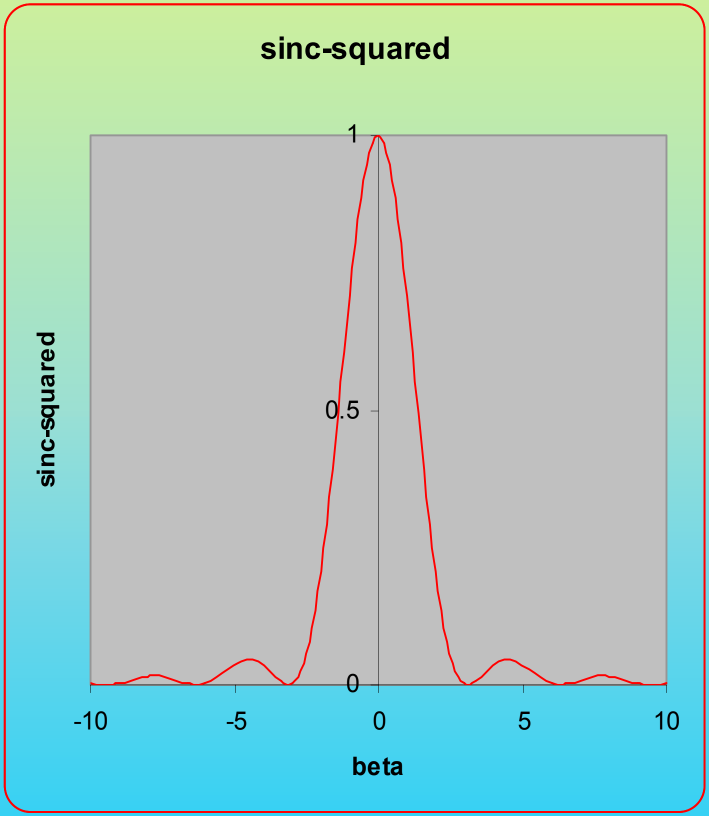
- The irradiance is therefore:

$$I = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$$

- There are zeros when:

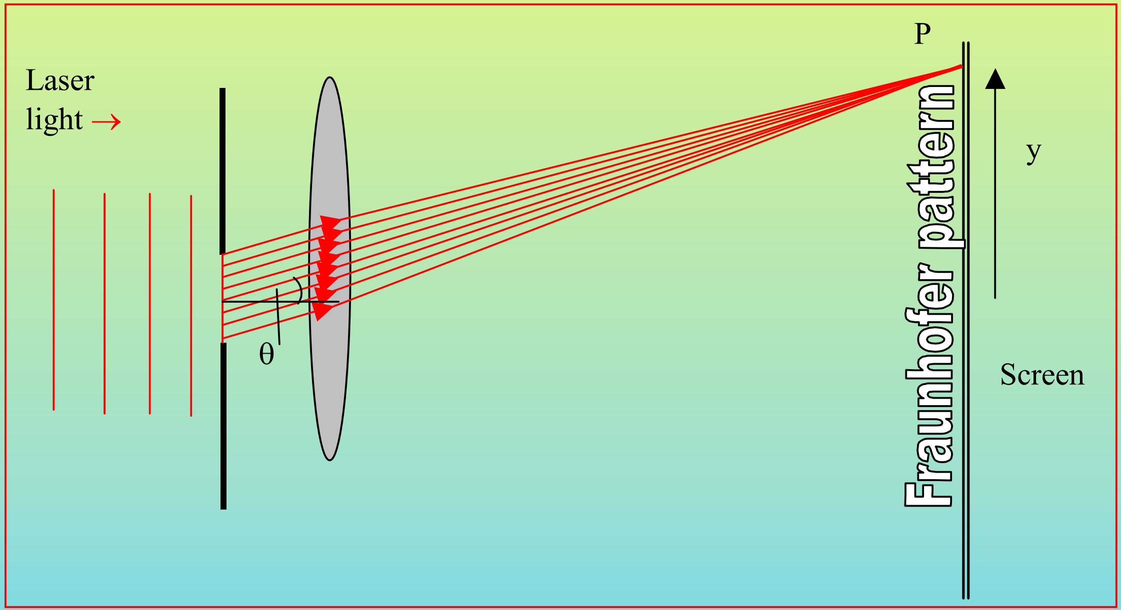
$$b\sin\theta = n\lambda$$

  - maxima are ~ half way between
  - the narrower the slit, the wider the pattern



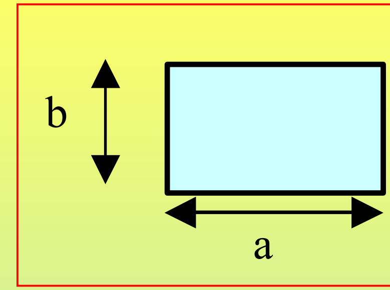
Making Fraunhofer diffraction happen

- It is easy to make the source a long way off, especially using a laser

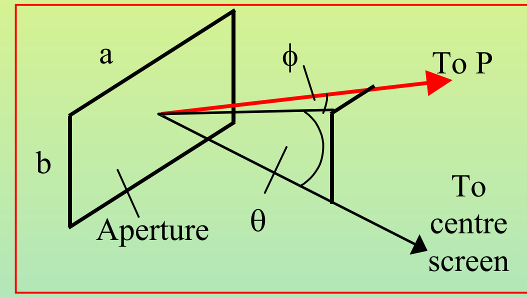


- A lens has the property that all parallel rays incident on it are converged in its rear focal plane
- The focal plane of a lens is therefore effectively 'at ∞' as far as formation of Fraunhofer diffraction is concerned
  - the same trick in reverse can be used to make the source 'at ∞' by placing a lens in front of the aperture and the source in the front focal plane

Rectangular aperture

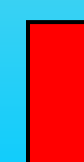


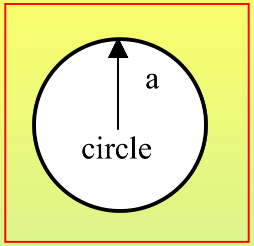
- A rectangular aperture has width *a* and height *b*
- The diffraction pattern varies as sinc^2 in two dimensions



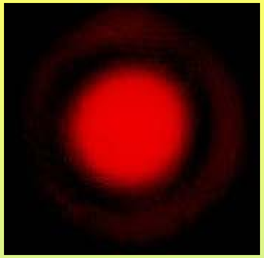
$$\alpha = k\frac{a}{2}\sin\phi$$

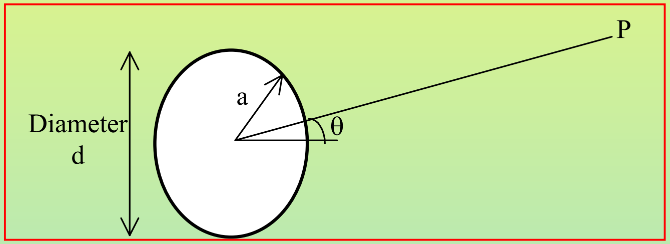
$$I = I(0)\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2$$

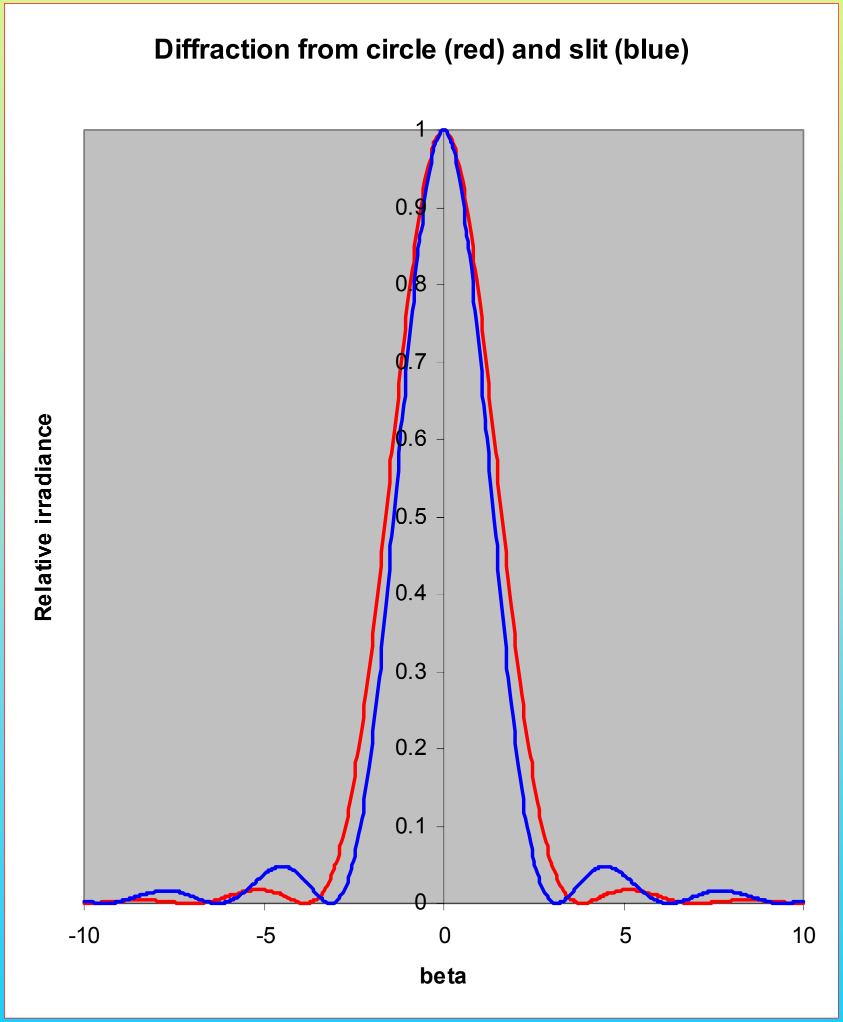
- The narrower either dimension, the wider the corresponding diffraction pattern
  - e.g. an aperture that looks like this: 
  - has a diffraction pattern made of rectangles roughly like this: 



Circular aperture





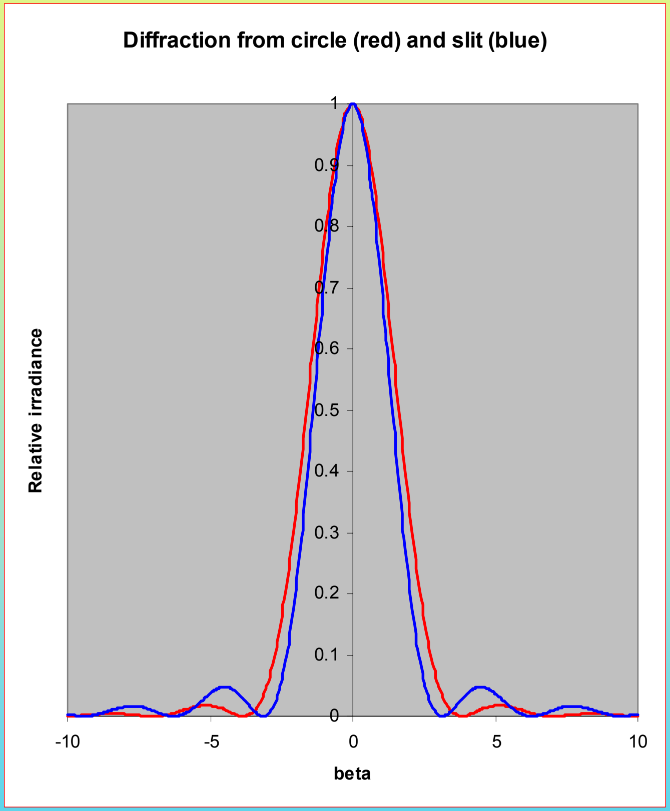


- The diffraction pattern consists of rings:–Airy ring pattern
- θ measures how far from the centre of the pattern you look
  - $\beta = ka\sin\theta$
- The figure compares the patterns of a slit that of the same width as the diameter of the circle

Mathematical detail

- The irradiance is given in terms of a Bessel function (as the mathematical might expect)

$$I = I(0)\left(2\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right)^2$$
$$= I(0)\left(2\frac{J_1(\beta)}{\beta}\right)^2 \text{, where } \beta = ka\sin\theta \;.$$



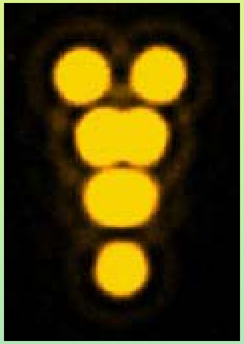
- The first zero, which determines the spread of the central region:

$$ka\sin\theta = 1.22\pi$$

  - the subsidiary maxima are smaller than those for a slit
  - the diffraction from the objective lens limits the resolution of observing instruments

Diffraction limited resolution

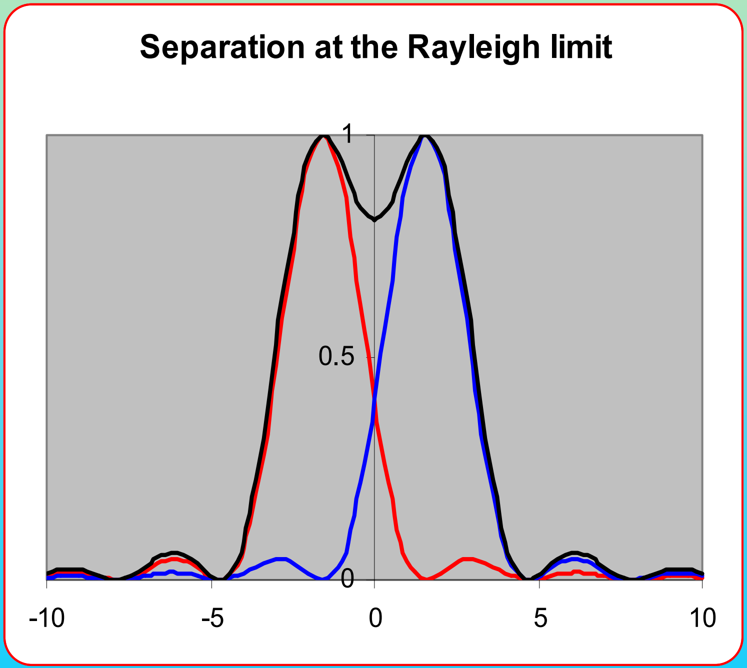
- Two closely spaced objects can just be resolved when the diffraction minimum of one lies on top of the maximum of its neighbour



- This is the Rayleigh criterion
- Numerically, the angular separation, $\Delta\theta$, of the two sources is therefore:

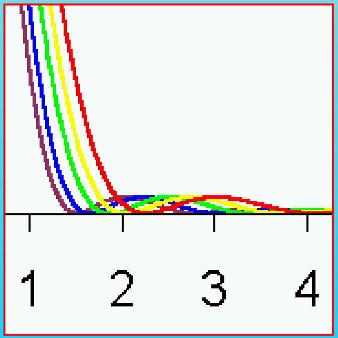
$$\Delta\theta = 1.22\,\lambda / d$$

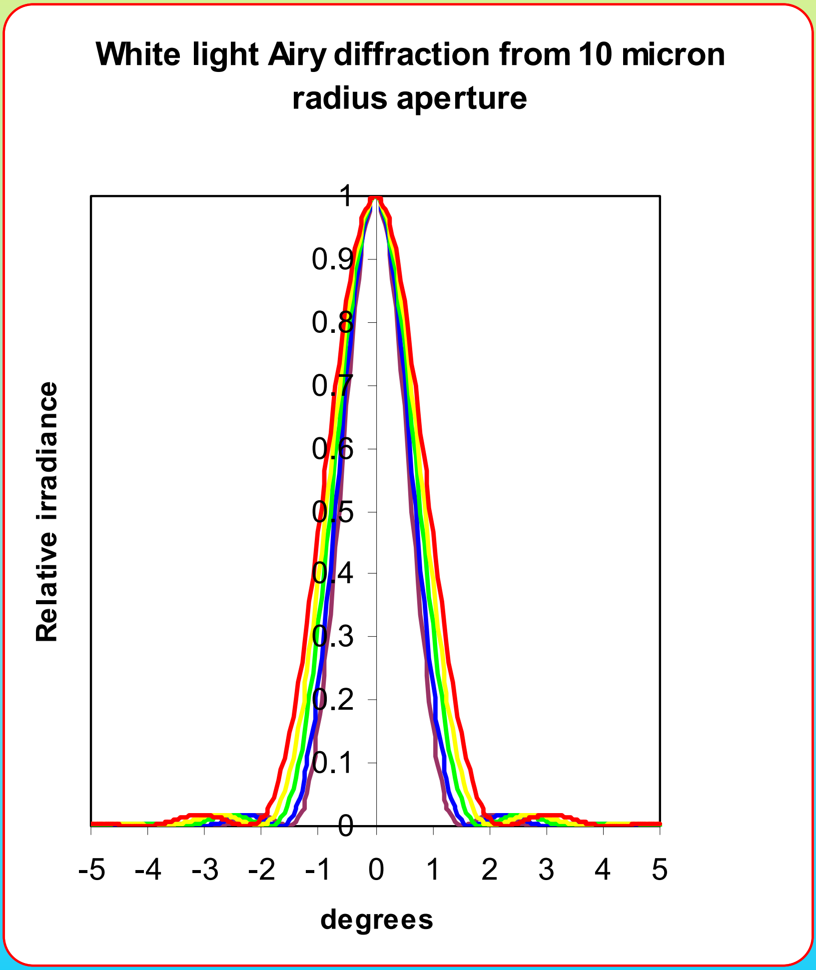
- e.g. d = 100 mm, $\Delta\theta$ = 1.2" arc when λ = 500 nm



White light pattern

- Centre of pattern is white
- Blue/violet → 0, leaving other half of spectrum
- Blue/violet rise to first maximum when red → 0
- Look at the enlargement ↓





Corona around the moon iridescent clouds

- The corona is frequently seen in Aberdeen, when altostratus clouds drift across the moon
- The corona is Airy's disk in the sky

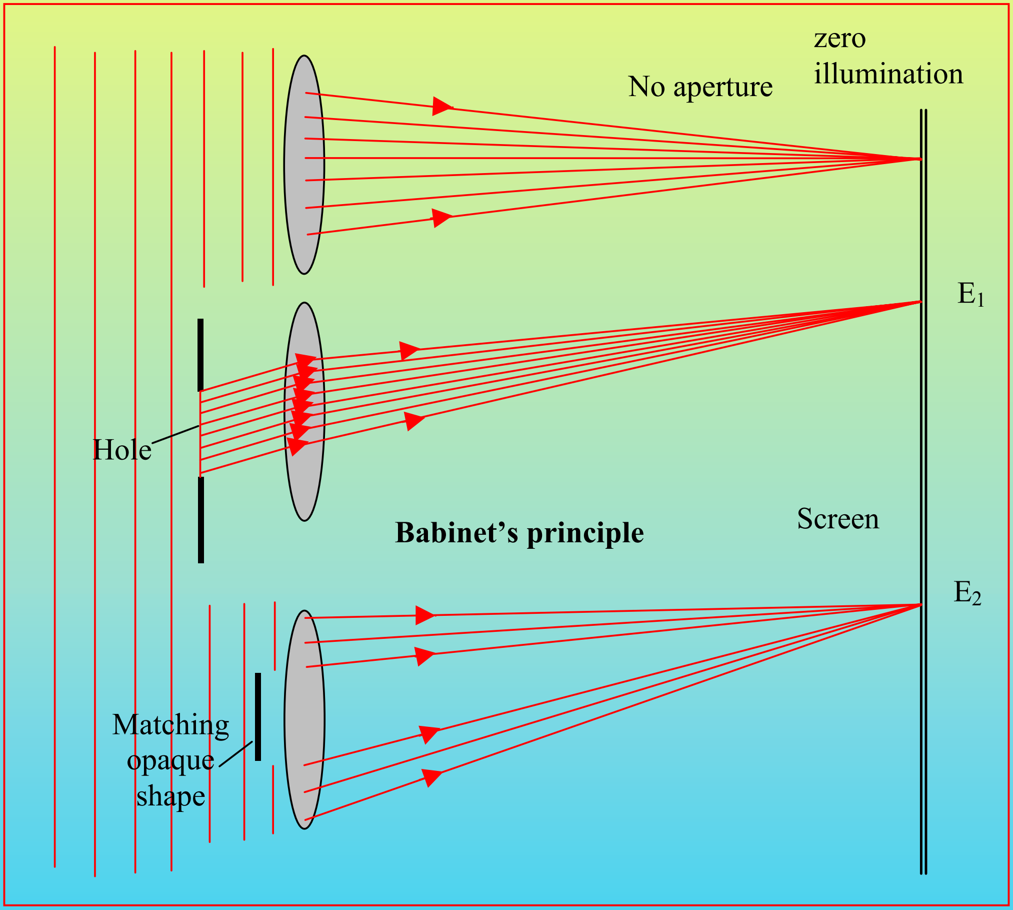
- Iridescent clouds are more irregularly coloured but cover a much larger area

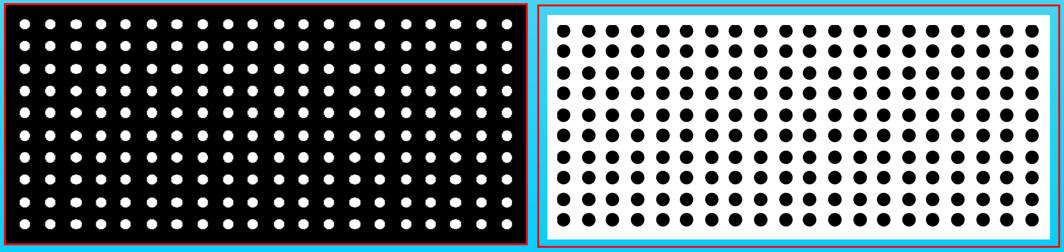
Babinet's principle

- Why is the diffraction pattern from drops the same as that from circular holes?
- Babinet's principle – based on:

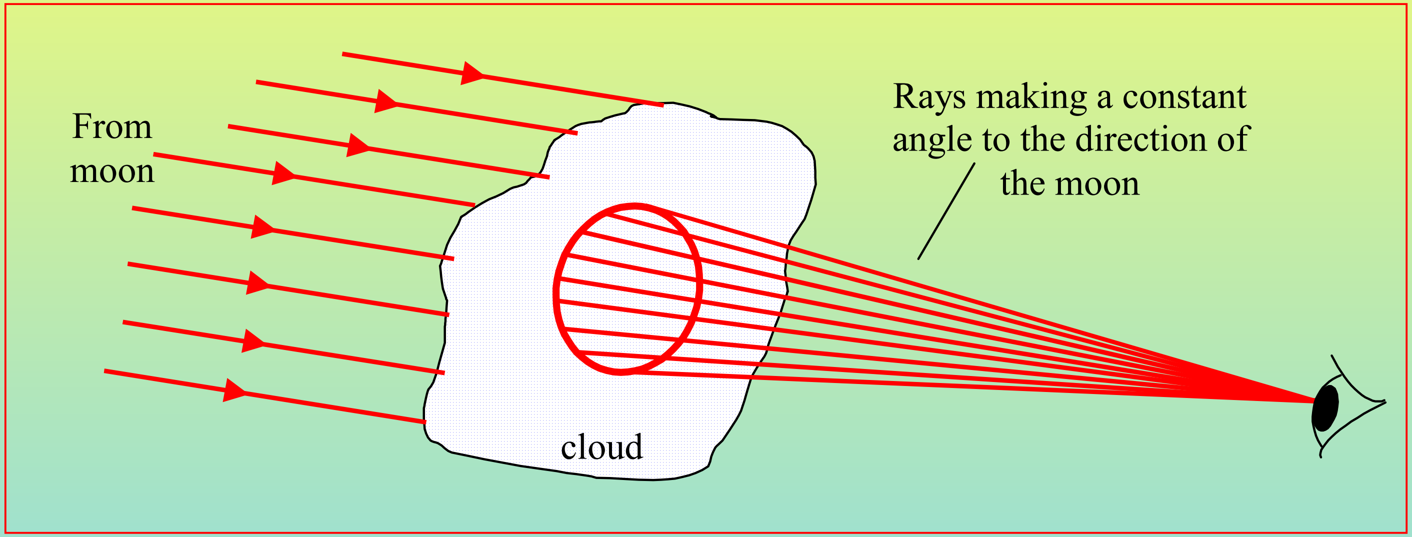
$$E_1 + E_2 = 0$$

  - i.e. $E_2 = -E_1$
- Result is that diffraction pattern from a hole is the same as that from a matching opaque shape

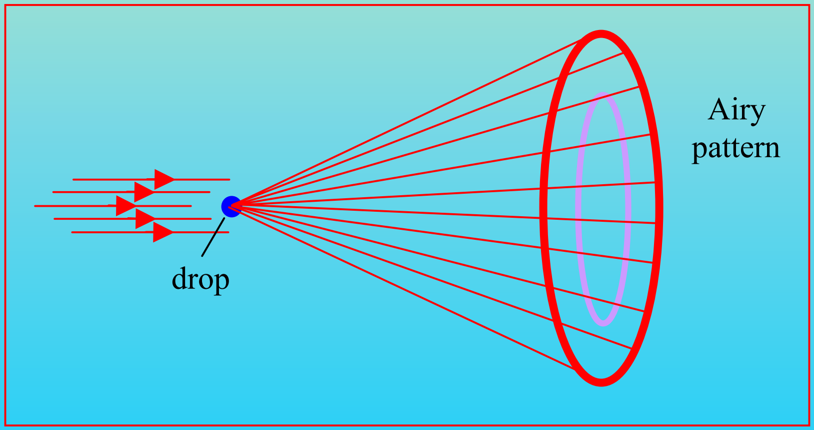




Viewing the cloud

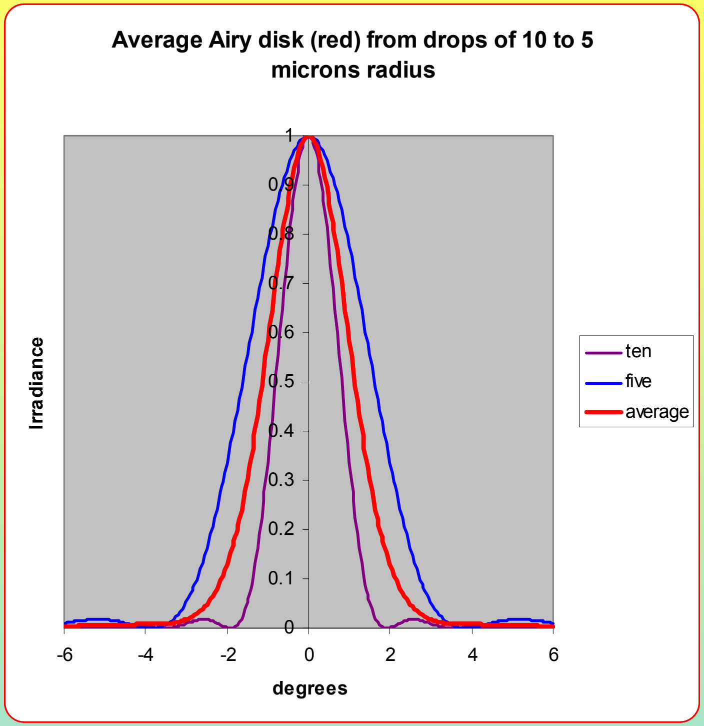


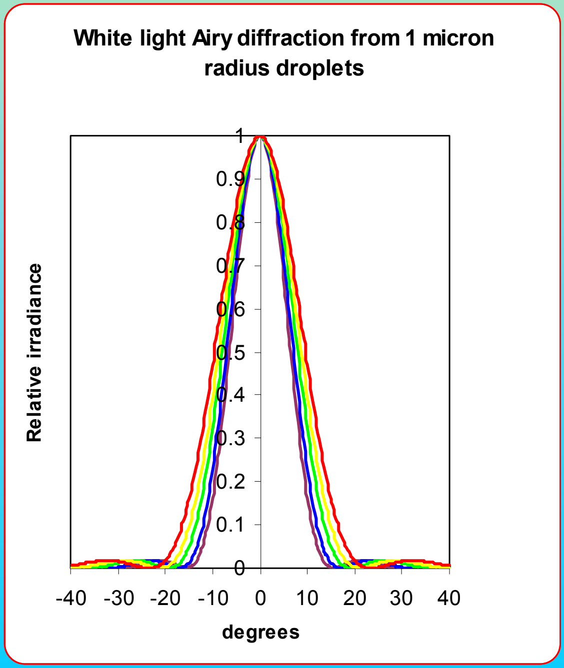
- All droplets at a fixed angle from the moon (e.g. 2°) lie on a circle
    - they radiate the same part of the Airy ring pattern to the observer
- The pattern seen is therefore the Airy disk



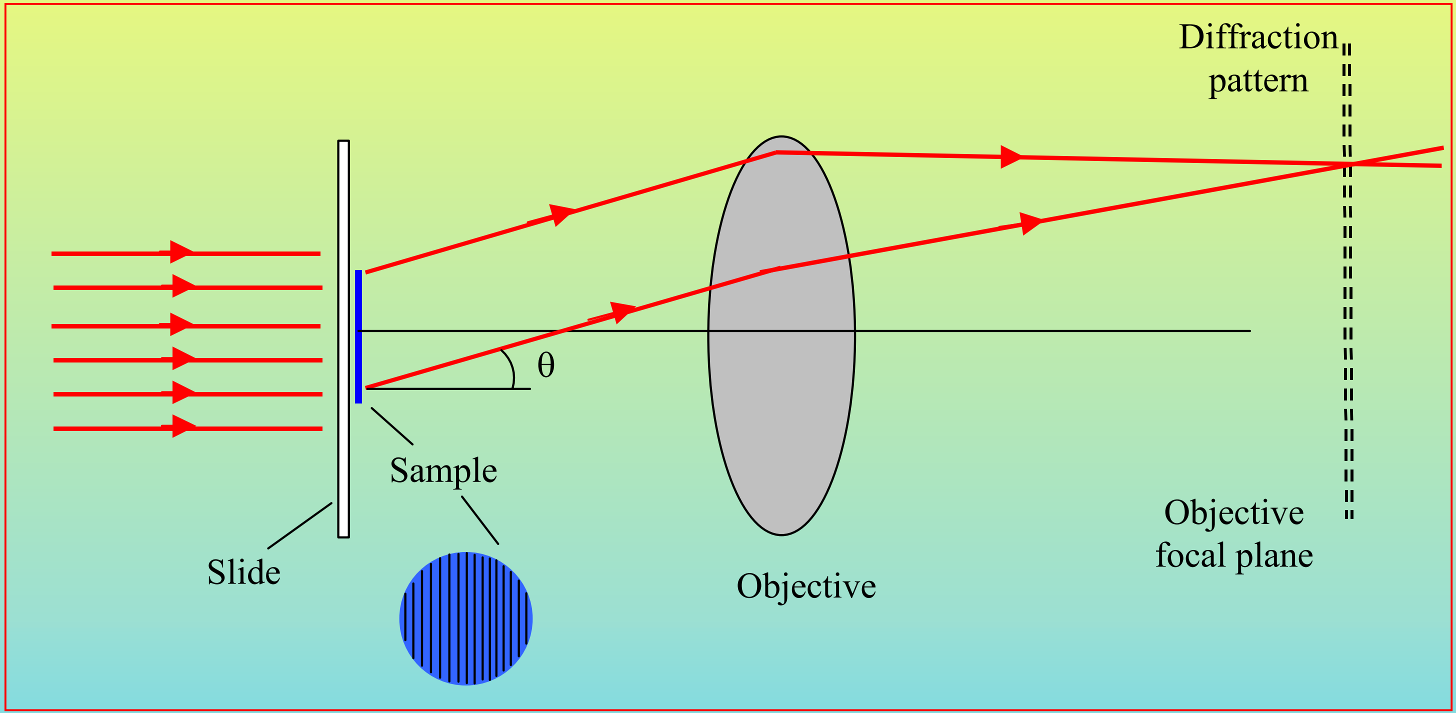
Additional points

- If you have a modest range of droplet sizes, only the first ring is visible
- If you have a wide range of droplet sizes, only a central aureole is visible
- iridescent clouds involve drops ~1 μm radius
- You can simply estimate the diameter of a uniform smear of regular particle on a slide by measuring the size of the diffraction ring
    - e.g. blood cells, spores, etc.





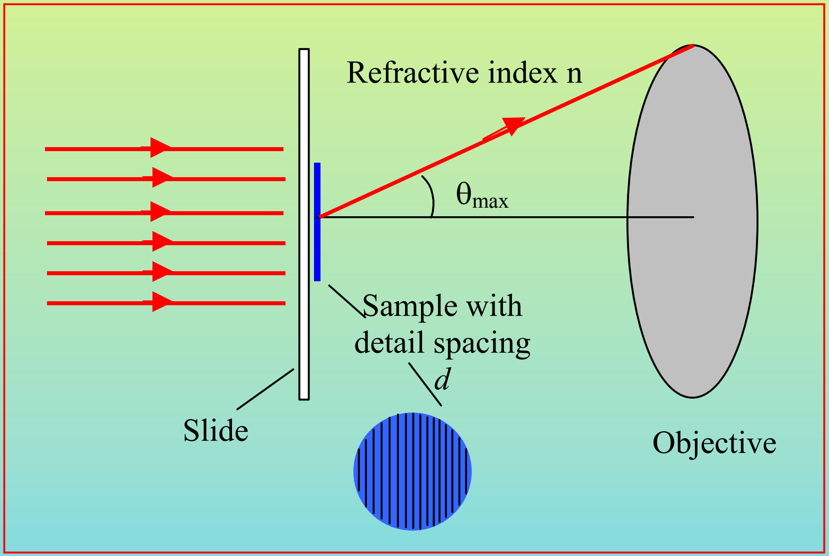
Diffraction and the microscope



- Fine detail on the object diffracts the incident light to the side
  - the finer the detail, the wider the diffraction
- This diffracted light must be picked up by the objective lens
- The objective lens gathering power therefore limits the fine detail that can be seen

Numerical aperture (NA)

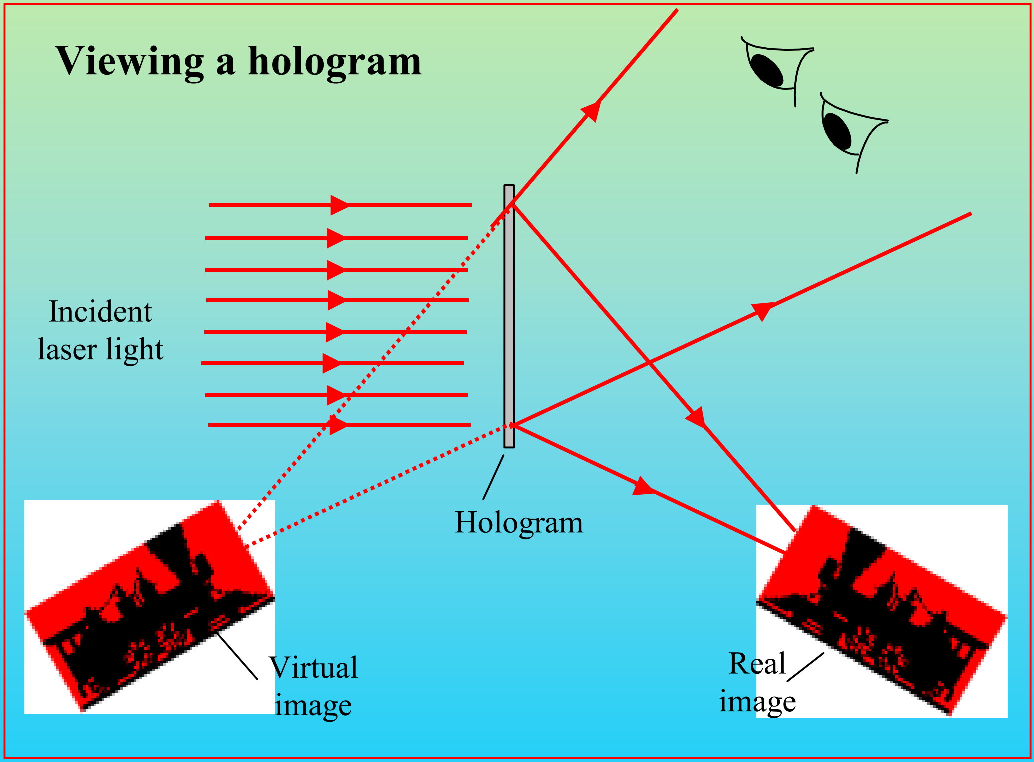
- The sample with spacing of detail *d* will diffract light to the side at an angle θ given by $d \sin\theta = \lambda$



- $NA = n \times \sin\theta_{max}$
  - n is usually 1.0
- The smallest detail that can be seen is determined by $d = \lambda / \sin\theta_{max} = \lambda/NA$
  - for detail, you need the largest NA possible
  - with $NA \approx 1$, $d \approx \lambda$

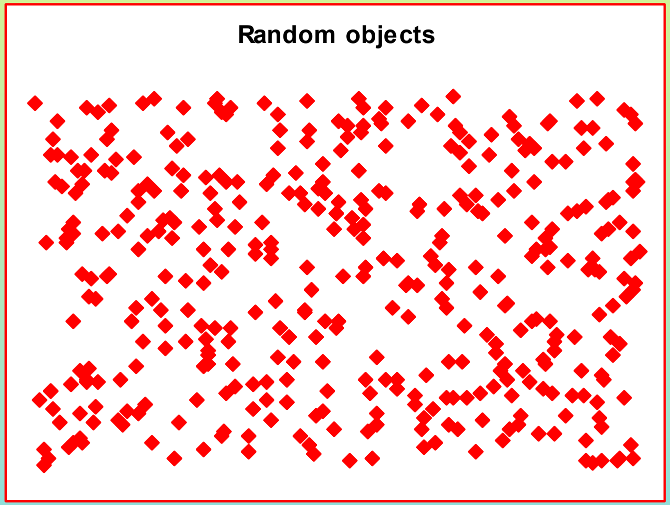
Viewing a hologram

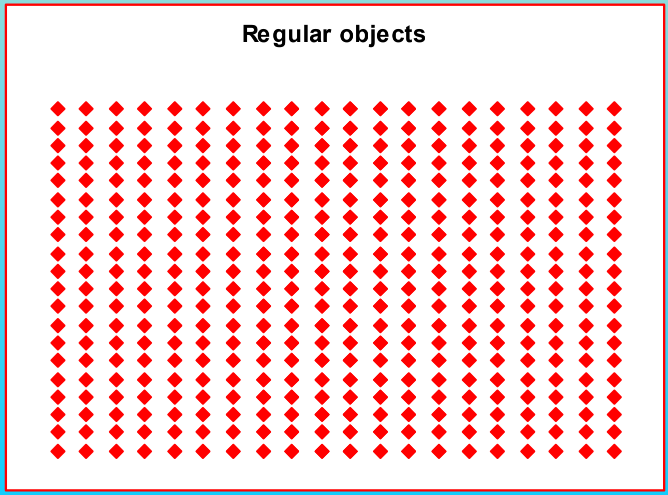
- The hologram diffracts, like a diffraction grating, when illuminated by laser light
- One diffraction order is seen as a virtual image, located where the original object was
- The reconstructed wavefront is just like the original wavefront coming from the object
- Our two eyes receive separate views of the object and let us visualise its 3D shape



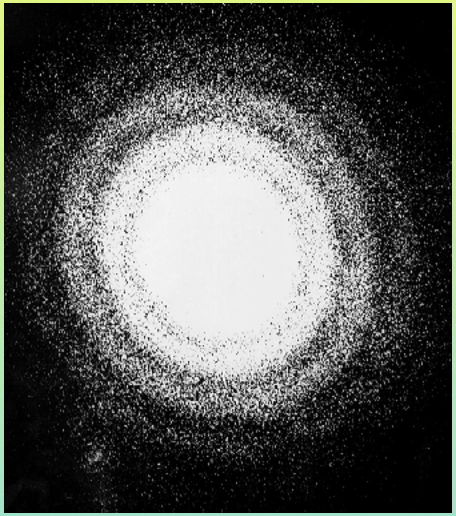
Fraunhofer diffraction from multiple apertures

- Multiple apertures are common
  - randomly distributed
  - regularly arranged
- The diffraction pattern from multiple identical apertures is the **product** of the **diffraction pattern of a single aperture** and the **interference pattern of a set of points** situated at the positions of each aperture in the pattern



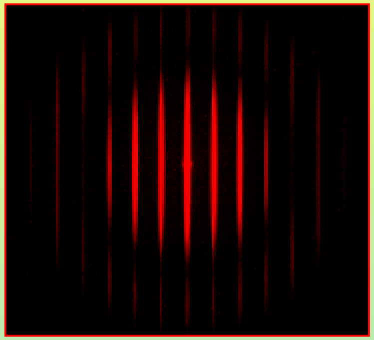


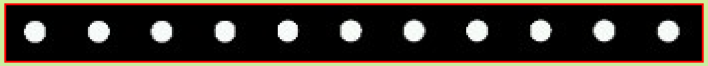
Random distribution of identical apertures

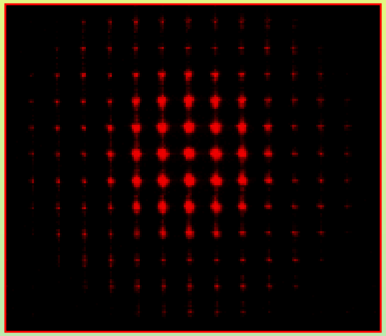


- The pattern seen is basically the same as the pattern from a single aperture
  - superimposed is a fine ‘spottiness’ whose detail depends on the random distribution
  - when the number of repetitions becomes very large, the spottiness isn’t seen
  - the irradiance increases as N, the number of repetitions in the pattern

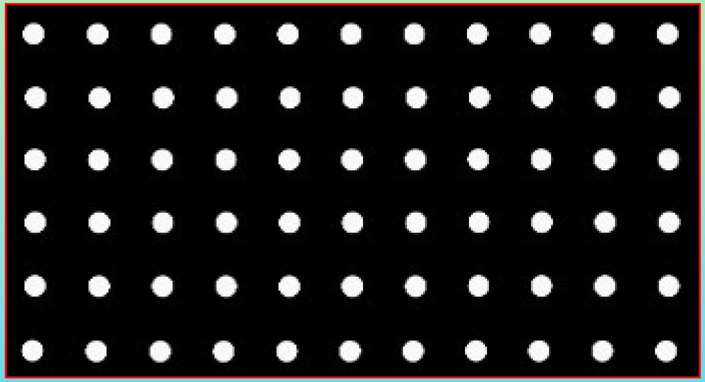
Regular distribution of identical apertures







- The diffraction pattern is the same as from the aperture but multiplied by the interference pattern for a set of points corresponding to the repetitions
  - here the repetitions are along a line
  - the irradiance is N^2 times the irradiance from a single aperture



- here the repetitions are in a square pattern, with equal spacing in two dimensions