Conjugate Ratio:
The ratio of object distance $s$ to image distance $s''$ along the principal axis of a lens or mirror. For an object at the focal point of a lens, the conjugate ratio is infinite.

For an object placed at twice the focal length of the lens or mirror, the image is formed at twice the focal length and the conjugate ratio is 1.

$$\frac{1}{s''} = \frac{1}{f} - \frac{1}{s}$$

$F = 50 \text{ mm}, s = 200 \text{ mm}, s'' = 66.7 \text{ mm}$

Conjugate Points:
The object point and image point of a lens system are said to be conjugate points. Since all the light paths from the object to the image are reversible, it follows that if the object were placed where the image is, an image would be formed at the original object position.
<table>
<thead>
<tr>
<th>Plano-convex lens</th>
<th>Plano-concave lens</th>
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<tbody>
<tr>
<td><img src="image1" alt="Plano-convex lens diagram" /></td>
<td><img src="image2" alt="Plano-concave lens diagram" /></td>
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<table>
<thead>
<tr>
<th>Bi-convex lens</th>
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<tr>
<td><img src="image3" alt="Bi-convex lens diagram" /></td>
<td><img src="image4" alt="Bi-concave lens diagram" /></td>
</tr>
<tr>
<td>Positive meniscus lens</td>
<td>Negative meniscus lens</td>
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<td><img src="image1.png" alt="Positive meniscus lens diagram" /></td>
<td><img src="image2.png" alt="Negative meniscus lens diagram" /></td>
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</table>
**F-Number (f/#):**  \( f/# = f/\phi \)

The f-number (also known as the focal ratio, relative aperture, or speed) of a lens system is defined to be the effective focal length divided by system clear aperture.

**Numerical Aperture (NA):**  \( NA = n \sin\theta \)

The numerical aperture of a lens system is defined to be the sine of the angle, \( \theta_1 \), that the marginal ray (the ray that exits the lens system at its outer edge) makes with the optical axis multiplied by the index of refraction (\( n \) ) of the medium.
Stops in Optical Systems

In any optical system, one is concerned with a number of things including:

1. The brightness of the image

Two lenses of the same focal length \((f)\), but diameter \((D)\) differs.

S

S’
Aperture Stop

- A stop is an opening (despite its name) in a series of lenses, mirrors, diaphragms, etc.

- The stop itself is the boundary of the lens or diaphragm

- Aperture stop: that element of the optical system that limits the cone of light from any particular object point on the axis of the system
Aperture Stop (AS)
Entrance Pupil ($E_nP$)

is defined to be the image of the aperture stop in all the lenses preceding it (i.e. to the left of AS - if light travels left to right)

How big does the aperture stop look to someone at O

$E' = E_nP$

$E_nP$ – defines the cone of rays accepted by the system
The exit pupil is the image of the aperture stop in the lenses coming after it (i.e. to the right of the AS).

Exit Pupil ($E_xP$)

$E'' = E_xP$
Chief Ray

- for each bundle of rays, the light ray which passes through the centre of the aperture stop is the chief ray
- after refraction, the chief ray must also pass through the centre of the exit and entrance pupils since they are conjugate to the aperture stop
- $E_nP$ and $E_xP$ are also conjugate planes of the complete system
Marginal Ray

• Those rays (for a given object point) that pass through the edge of the entrance and exit pupils (and aperture stop).
Field Stop

- That component of the optical system that limits the field of view

\[ A = \text{angular field of view} \]

\[ \theta = \text{angular field of view} \]

\[ A = \text{field of view at distance } d \]
Optical Aberrations and Diffraction

An ideal lens will focus an input parallel beam to a perfect point (focal point). The size of the focal point should be infinitesimal. However, because of lens aberrations and diffraction, the focal spot of a real lens has a finite size. The size of the focal spot is a measure of lens aberrations and diffraction. All singlet lenses have significant amount of aberrations.
Optical Aberrations

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

The sine functions in Snell's law can be expanded in an infinite Taylor series:

\[ \sin \theta_1 = \theta_1 - \theta_1^3 / 3! + \theta_1^5 / 5! - \theta_1^7 / 7! + \theta_1^9 / 9! - \ldots \]

The first approximation we can make is to replace all sine functions with their arguments (i.e., replace \( \sin \theta_1 \) with \( \theta_1 \) itself and so on). This is called first-order or paraxial theory because only the first terms of the sine expansions are used. Design of any optical system starts with this approximation.

The assumption that \( \sin \theta = \theta \) is reasonably valid for \( \theta \) close to zero (i.e., high f-number lenses).

With more highly curved surfaces (and particularly marginal rays), paraxial theory yields increasingly large deviations from real performance because \( \sin \theta \neq \theta \). These deviations are known as aberrations.
Seidel developed a method of calculating aberrations resulting from the $\theta^3/3!$ term. The resultant third-order lens aberrations are therefore called *Seidel aberrations*.

Seidel put the aberrations of an optical system into several different classifications:

- In monochromatic light they are *spherical aberration, astigmatism, field curvature, coma, and distortion*.
- In polychromatic light there are also *chromatic aberration and lateral color*.

In practice, *aberrations occur in combinations rather than alone*. This system of classifying them, which makes analysis much simpler, gives a good description of optical system image quality.
Focusing with aberration-free lens

Focusing with a lens exhibiting spherical aberration

The distance along the optical axis between the intercept of the rays that are nearly on the optical axis (paraxial rays) and the rays that go through the edge of the lens (marginal rays) is called longitudinal spherical aberration (LSA).

The height at which these rays intercept the paraxial focal plane is called transverse spherical aberration (TSA). These quantities are related by

\[ TSA = LSA \times \tan \theta \]

spot size due to spherical aberration = \( \frac{0.067 f}{f/\#^3} \)
Astigmatism
Coma
Field Curvature

Distortion

OBJECT

PINCUSHION DISTORTION

BARREL DISTORTION
Chromatic aberrations

Logitudinal chromatic aberration

Lateral Color
Aberrations of positive singlets as a function of shape for infinite conjugate ratio

Coddington shape factor, $q$, defined as $q = \frac{r_2 + r_1}{r_2 - r_1}$
Three possible systems for use at the unit conjugate ratio
Cooke Triplet Lenses

All above lens configurations provide improved performance on-axis only. To achieve good performance both on- and off-axis, more complex lens forms are required. Cooke triplet is a well-known lens form that provides good imaging performance over a field of view of +/- 20-25 degrees. Many consumer grade film cameras use lenses of this type.